I.4 Damped simple harmonic motion

If there is ………… to motion then the spring / mass (or pendulum) will gradually lose …………..

To model this, we have to introduce a …………… ………. into our SHM analysis.

The retarding force (in this analysis) is proportional to the velocity

(b is a constant)

Balancing forces;

So, for damped motion, we have the equation of motion;

$$m\ddot{x} + b\dot{x} + kx = 0$$

To solve it, we <u>try</u> a solution of the form;

(C will have the dimensions of length, α of inverse time.)

Differentiate w.r.t time;

Then ………….. into top equation;

Simplifying;

The solution C = 0 is trivial;

Therefore, we look for solutions to

We can use the standard …………. equation for this;

And when simplified slightly;

Therefore, the solution to the equation of motion for this damped oscillator is;

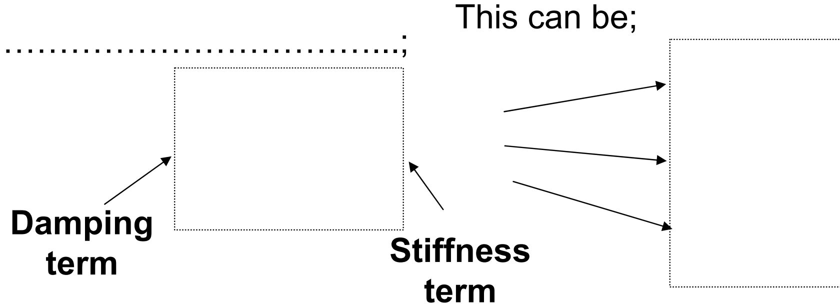
Or, slightly more simply,

We find this solution for x gives us three regimes of physical motion.

$$x = Ce^{-\frac{b}{2m}t}.e^{\left(\pm\sqrt{\frac{b^2}{4m^2}-\frac{k}{m}}\right)t}$$

The problem …………………………..
……………………………………………..;

This can be;



i) the system is …………………..…; the damping term dominates over the stiffness term and a …………………….. motion results.

ii) ……………………; when the damping term and stiffness term are equal, the system returns to zero amplitude in ……………………………

iii) the system is ………………………… and gives oscillatory (simple harmonic) but …………………………………………

UNIVERSITY of EXETER

Dr. Pete Vukusic, Exeter University: PHY 1106: Waves and Oscillators (Lecture 4)

Case i): Heavy damping (damping term dominates over stiffness term)

(Pain, p31).

This leads to …………………..… motion;

The equation to predict the displacement of our object is;

$$x = Ce^{-\frac{b}{2m}t}.e^{\left(\pm\sqrt{\frac{b^2}{4m^2}-\frac{k}{m}}\right)t}$$

Under certain boundary conditions, it can be re-expressed as;

$$x = C'e^{-\frac{b}{2m}t}\sinh\left(\left(\frac{b^2}{4m^2}-\frac{k}{m}\right)^{\frac{1}{2}}t\right)$$

(Pain, p31).

Displacement

time

This illustrates the non-oscillatory behavior of a ………. ……….. oscillator.

5

UNIVERSITY of EXETER

Dr. Pete Vukusic, Exeter University: PHY 1106: Waves and Oscillators (Lecture 4)

Case ii): Critical damping (damping term equals stiffness term)

(See Pain, p32).

Again this case leads to ……………………. motion;

This time, instead of our original equation;

$$x = Ce^{-\frac{b}{2m}t}.e^{\left(\pm\sqrt{\frac{b^2}{4m^2}-\frac{k}{m}}\right)t}$$

We can re-express it as; $x = (A + Bt)e^{-\frac{bt}{2m}}$

(since α has equal roots Pain, p32).

A ………………….. oscillator returns to equilibrium …………………………….

Displacement

time

(albeit very damped oscillation).

6

Case iii): Light damping (stiffness term dominates over damping term)

This case leads to ……………. damped SHM

The most common examples occur for this last type of case; i.e. when the damping is light.

e.g. ……………………………………………………………..

This case demands a rigorous analysis…..

But, there is a problem!

So rewrite as;

Therefore the solution to the equation of motion for this case is;

Or for simplicity;

Where;

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

→ **ie. the ……………. is modified because of the …………….. !**

can be written as

If we take the ……… …… of this (which is the …………. …………. part), then we get the solution of the form;

To allow for motion starting anywhere in the cycle, we ………………………….. (ϕ)

(Pain p33).

so re- write as; $$x = Ce^{-\frac{bt}{2m}}\left(\cos\omega' t + \phi\right)$$

Equally, the solution could be written as;