X-ray Interaction (Part I)

Possible Interactions
• Three types of interaction
  1. Scattering
     - Interaction with an atom
     - Deflected
     - May or may not loss of energy
  2. Absorption
     - Loses all of it energy to the atom
  3. No Interaction
     - Passes through material without interacting ANY atoms

Possible Interactions
• Compare radiation intensity before and after interaction with medium
  • Lower intensity after passing through medium
  • Beam has been attenuated by processes of scattering & absorption

Probability & Cross-section
• Probability of X-ray photon interacting with a particular atom is low
  • However, very large number of atoms in a small volume of solid increase the probability of an interaction
Probability & Cross-section

• X-ray beam of area $A$ incident on medium with atoms of cross-sectional area $a$
• If photon hits atom it is either absorbed or scattered from primary beam
• Cross-section:
  - Apparent Area, $a$, NOT the actual area
  - Area likely in interact with X-ray
  - Depends upon $Z$ & photon energy
  - Typically $1.5 \times 10^{-28}$ m$^2$

Total Linear Attenuation Coefficient ($\mu$)

• Parallel beam of monoenergetic photons will undergo exponential attenuation as it passes through a uniform medium
  \[ I_x = I_0 \exp(-\mu x) \]
• $\mu$ is the total linear attenuation coefficient
• Definition:
  - Fraction of X-rays removed from beam per unit thickness of material
• Increases as probability of interaction ($aN_x$) increases

Total Mass Attenuation Coefficient ($\mu/\rho$)

• Probability of interaction proportional to number of atoms per unit volume, $N$
• If medium is heated volume will increase and $N$ will decrease
• Example: Double thickness & half density
• $\mu/\rho$ will be unchanged

Total Attenuation Coefficient

• Total attenuation coefficient is the sum of the attenuation coefficients due to each attenuation process
  - I.e. total linear attenuation coefficient is the sum of the individual linear attenuation coefficients
  - I.e. total mass attenuation coefficient is the sum of the individual mass attenuation coefficients

Linear Attenuation Coefficients

- $\mu = \tau + \sigma + \pi$
  - $\tau$ linear attenuation coefficient due to photoelectric effect
  - $\sigma$ linear attenuation coefficient due to Scattering
  - $\pi$ linear attenuation coefficient due to Pair Production

Mass Attenuation Coefficients

- $\mu/\rho = \tau/\rho + \sigma/\rho + \pi/\rho$
**Total Attenuation Coefficient**

- Attenuation is the sum of absorption & scattering

\[ \mu = \tau + \sigma + \pi \]

<table>
<thead>
<tr>
<th>Photoelectric</th>
<th>Compton &amp; Rayleigh</th>
<th>Pair Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau = \tau_a + \tau_s )</td>
<td>( \sigma = \sigma_a + \sigma_s )</td>
<td>( \pi = \pi_a + \pi_s )</td>
</tr>
</tbody>
</table>

**Attenuation Processes**

Five x-ray attenuation processes:
1. Coherent (or Elastic) Scattering
2. Photoelectric Effect
3. Compton Scattering
4. Pair Production
5. Photodisintegration

**Coherent Scatter**

- If the energy of a photon is considerably less than binding energies of orbiting electrons of an atom the photon may be deflected from its path with no loss in energy
- Also called Classical or Rayleigh Scattering

**Coherent Scatter**

- Predominantly forward scatter
- Elastic scatter can not occur if recoil experienced by atom is sufficient to cause excitation or ionization
- No absorption: No energy has been permanently transferred to material

**Coherent Scatter**

- Low attenuation
- Photons scattered through small angle
- Particularly when \( E > 100 \text{eV} \) & low \( Z \)
- Contribution to mass attenuation:

\[ \frac{\sigma_{coh}}{\rho} \propto \frac{Z^2}{E} \]
**Photoelectric Effect**

- X-ray photon involved in an inelastic collision with an orbiting electron
- Photon gives up ALL of its energy and therefore disappears (absorbed)
- Electron is ejected from atom
- Absorption can only take place if photon energy is equal to or greater than electron binding energy

**Photoelectric Effect**

- X-ray photon of energy $\nu$
- Electron ejected from K-shell
- Some of the photon energy is used in overcoming electron binding energy, $B$
- Remaining energy is given to electron as kinetic energy
- Electron Kinetic Energy $= (\nu - B)$

**Photoelectric Effect**

- Vacancy created in K-shell will be filled by electron from the L-shell
- Quantum 'jumps' producing characteristic radiation
- Energy of characteristic photon is equal to energy difference between shells
- For tissue, energy difference is very small
  - $(1.2 - 1.8 \times 10^{-2} \text{ eV}) \Rightarrow \text{Infrared}$

**Photoelectric Effect**

- Probability of photoelectric interaction occurring at a particular shell depends upon binding energy and photon energy
- Zero when $\nu < B$
- Greatest when $\nu = B$
- Decreases rapidly with photon energy
  - When $\nu > B$: $\frac{1}{E^3}$ term approximates to $E^2$ & eventually $E$

**Photoelectric Effect and Attenuation Coefficient**

- Mass attenuation coefficient is related to the atomic number of the absorber ($Z$) and the photon energy ($E$)
- Approximated by: $\frac{T}{\rho} \propto \frac{Z^3}{E^3}$
- Applies to $E$ up to 200 keV.
- A higher energies $E^3$ term approximates to $E^2$ & eventually $E$

**Photoelectric Effect**

- Photoelectric Effect causes both attenuation and absorption, BUT NOT scattering
- Individual photons are removed from beam
  - Attenuation
- Energy is imparted to the absorbing medium
  - Absorption
- Energy absorbed
  - Kinetic energy of ejected photon
  - Energy of recoil of absorbing atom
  - Energy of characteristic radiation
**Photoelectric Effect**

*Contribution of photoelectric effect to the radiographic image*

- Need to consider the linear attenuation coefficient:
  - For a given photon energy attenuation is proportional to density of tissue and atomic number cubed
- Bone is approximately twice as dense as soft tissue with an atomic number twice that of tissue
  - Therefore linear attenuation coefficient of bone is approximately 16 times greater than that of soft tissue

\[ \tau \propto \frac{\rho Z^3}{E^3} \]

**Compton Scattering**

- If photon energy is much higher than electron binding energy, electron may be considered as a free electron
- Interaction between free electron and photon is *Compton Scattering*
- Partial absorption of photon energy

**Compton Scattering**

- Photon may be scattered in any direction
- Electron can only travel forwards relative to incident photon
- Partial absorption of photon energy

\[ \lambda_2 - \lambda_1 = \frac{h(1 - \cos \theta)}{mc} \]

**Compton Scattering and Attenuation Coefficient**

- Probability of Compton scattering occurring per unit mass is proportional to the density of electrons and inversely proportional to the photon energy

\[ \frac{\sigma}{\rho} \propto \frac{\text{electron density}}{E} \]

**Electron Density**

- Can use Avogadro’s number [Clake Section 5.6] to calculate the number of atoms per mole of an element of mass number (A)
  \[ \text{atoms per unit mass} = \frac{N_a}{A} \]
- Number of electrons (in a normal atom) is equal to the number of protons (Z)
  \[ \text{electron density} = N_a \times \frac{Z}{A} \]

**Electron Density**

- Assume that most elements has equal numbers of protons and neutrons
  - \( Z/A = 0.5 \)
- Hydrogen – 6 X \( 10^{23} \) electrons per kg
- Everything else – 2.5-3.5 X \( 10^{23} \) electrons per kg
Compton Scattering

Mass Attenuation Coefficient \((\sigma/\rho)\)
- Inversely proportional to photon energy
- Measure of the total energy removed from the beam
- Sum of scattering & absorption coefficients
  \[ \sigma = \sigma_a + \sigma_s \]
  \[ \frac{\sigma}{\rho} = \frac{\sigma_a}{\rho} + \frac{\sigma_s}{\rho} \]

Compton Scattering

Mass Absorption Coefficient \((\sigma_a/\rho)\)
- Represents the fraction of the total energy removed from the medium
- Higher photon energy \(\Rightarrow\) higher energy loss
  - \(\sigma_a/\rho\) and \(\sigma_s/\rho\) are closer together for higher energies

Compton Scattering

Mass Scattering Coefficient \((\sigma_s/\rho)\)
- Represents the fraction of energy not removed from the beam

Pair Production

- Formation of two charged particles from a single high-energy photon
- Can only occur for photon energies greater than 1.02 MeV
  - (Equivalent to twice the rest mass of an electron)
- Produces electron and positron pair

Pair Production

\[ E = \left( m_0 c^2 + T_1 \right) + \left( m_0 c^2 + T_2 \right) \]
- \(E\) is the photon energy
- \(m_0\) is electron (or positron) rest mass
- \(c\) is the speed of light
- \(T_1\) & \(T_2\) are the KE of the electron and positron respectively
Pair Production

$E = 2m_0c^2 + T_1 + T_2$

- $E$ is the photon energy
- $m_0$ is electron (or positron) rest mass
- $c$ is the speed of light
- $T_1$ & $T_2$ are the KE of the electron and positron respectively

Pair Production

Incident Photon $E > 1.02$ MeV

$E = 1.02 + T_1 + T_2$,

- $E$ is the photon energy
- $m_0$ is electron (or positron) rest mass
- $c$ is the speed of light
- $T_1$ & $T_2$ are the KE of the electron and positron respectively

Pair Production

Attenuation, Absorption & Scattering

- Attenuation related to both Photon energy & atomic number
  \[ \frac{\pi}{\rho} = (E - 1.02)Z \]
- KE of electrons & positrons are absorbed by the medium
- Energy absorbed is less than original photon energy
  - $(E - 1.02)$ MeV

Pair Production

- Kinetic Energy of electrons & positrons is absorbed by the medium
- Energy absorbed is less than original photon energy
  - $(E - 1.02)$ MeV
- Electron will eventually lose all its energy to the medium
- Positron will eventually collide with an electron
  - Positron-electron annihilation
  - Producing two photons each with energy 0.51 MeV

Pair Production

- If the two photons of annihilation radiation are absorbed by the medium, then the total energy absorbed is:
  \[ \text{Energy Absorbed} = (E - 1.02) + (2 \times 0.51) \]
- I.e. All the original photon energy has been absorbed
- This doesn't always happen!
- In such case absorption coefficient $(\pi_a < \pi)$ by fraction $(E - 1.02)/E$

Pair Production

- If the two particles of annihilation radiation are absorbed by the medium, then the total energy absorbed is:
  \[ E_{\text{absorbed}} = (E - 1.02) + (2 \times 0.51) \]
- I.e. All the original photon energy has been absorbed
- This doesn't always happen!
- In such case absorption coefficient $(\pi_a < \pi)$ by fraction:
  \[ \frac{(E - 1.02)}{E} \text{ or } 1 - \frac{1.02}{E} \Rightarrow \pi_a = \frac{\pi(1 - 1.02)}{E} \]
Pair Production

- As with the previous attenuation processes:
  \[ \pi = \pi_a + \pi_p \quad \text{and} \quad \frac{\pi}{\rho} = \frac{\pi_a}{\rho} + \frac{\pi_p}{\rho} \]
- \( \pi_a \) is the fraction of energy carried by the two annihilation photons (each of energy 0.51MeV)
- For both diagnostic & therapeutic energies \( \pi_a \) can be ignored

\[ \pi = \pi_a \quad \text{and} \quad \frac{\pi}{\rho} = \frac{\pi_a}{\rho} \]

Photodisintegration

- Very-high-energy photons (>10MeV) can escape interaction with electrons and nuclear electric field.

\[ \text{Incident Photon} \]
\[ E > 10 \text{ MeV} \]
\[ \text{Nuclear fragment} \]

Summary

- Photoelectric effect dominates at low energies (50-500keV)
- Absorption edges are more pronounced for elements with larger Z
- Compton Scattering dominates over a wider range (50keV - 5MeV)
- Compton attenuation is independent of material (with constant density)
- Pair production is only significant for very high energies (>1.02 MeV) and materials with high atomic number (Z)

X-ray Interaction 1 - Problem Sheet

1. When kV is increased, is there an increase or decrease in Compton scattering? Explain your answer
2. A 60 keV x-ray photon ionizes a Barium atom by ejecting an O-shell electron with 12 keV of kinetic energy. What is the energy of the Compton scattered x-ray photon (the binding energy of an O-shell electron is 0.04keV)
3. The energy of the Compton-scattered x-ray is equal to the difference of what two energies?
4. Of the five basic mechanisms of x-ray interactions with matter, which are not important to diagnostic radiography and explain why?
5. A 30 keV x-ray interacts photoelectrically with a K-shell electron of a calcium atom. What is the kinetic energy of the Compton electron? (the k-shell electron binding energy of Calcium is 4 keV)
6. (a) How much less likely will an interaction be for a 50 keV x-ray photon with soft tissue than for a 20 keV photon? (b) How much more likely is interaction with iodine than with soft tissue for a 70 keV photon?