Dielectric Tensors and Constants

Real dielectrics can be very complicated. A general expression for the components of the polarisation $P$ at a point where the field is $E$ is

$$P_i = P_{0i} + \varepsilon_0 \sum_j \chi^{(1)}_{ij} E_j + \varepsilon_0^2 \sum_{jk} \chi^{(2)}_{ijk} E_j E_k + \varepsilon_0^3 \sum_{jkl} \chi^{(3)}_{ijkl} E_j E_k E_l + \cdots.$$  

An isotropic material has off-diagonal elements equal to zero, diagonal elements that are equal to each other and no spontaneous polarisation $P_{0i}$. In this special case the above expression reduces to a simple power-series expansion

$$P_i = \varepsilon_0 \chi^{(1)} E_i + \varepsilon_0^2 \chi^{(2)} E_i^2 + \cdots$$

where $\chi^{(n)} = \chi^{(n)}_{11} = \chi^{(n)}_{22} = \chi^{(n)}_{33}$. The terms of order $E^2$ and above can be neglected for many materials in moderate fields in which circumstances

$$P = \varepsilon_0 \chi E$$

where the scalar constant $\chi$ is called the linear dielectric susceptibility of the material. If this is an adequate approximation then

$$D = \varepsilon_0 E + P = \varepsilon_0 (1 + \chi) E = \varepsilon E$$

and $\varepsilon$ is the permittivity of the dielectric. The abbreviation LIH (linear, isotropic and homogeneous) is often used in this context. The dielectric constant $\kappa$ is defined by

$$\kappa = \frac{\varepsilon}{\varepsilon_0} = 1 + \chi = \varepsilon_r \quad \text{(the relative permittivity)}.$$  

All these “constants” are frequency dependent to some extent because it takes a finite time for the dipoles to respond to the electric field.

Anisotropic materials which exhibit a large spontaneous polarisation $P_{0i}$ are known as ferroelectrics. A permanently polarised ferroelectric sample is sometimes called an electret (there is an analogy with a magnet). BaTiO$_3$ is an excellent example of a ferroelectric material. Related effects include changes in polarisation due to mechanical distortion of this crystal, piezoelectricity, and due to temperature changes, pyroelectricity.