Nanoscale Electromagnetic Compatibility: Quantum Coupling and Matching in Nanocircuits

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Abstract—The paper investigates two typical electromagnetic compatibility (EMC) problems, namely, coupling and matching in nanoscale circuits composed of nano-interconnects and quantum devices in entangled state. Nano-interconnects under consideration are implemented by using carbon nanotubes or metallic nanowires, while quantum devices - by semiconductor quantum dots. Equivalent circuits of such nanocircuits contain additional elements arising at nanoscale due to quantum effects. As a result, the notions of coupling and impedance matching are reconsidered. Two examples are studied: in the first one, electromagnetically coupled nanowires are connected to classical lumped devices; in the second one, electromagnetically uncoupled transmission lines are terminated on quantum devices in entangled states. In both circuits the EMC features qualitatively and quantitatively differ from their classical analogs. In the second example, we demonstrate the existence of quantum coupling, due to the entanglement, which exists in spite of the absence of classical electromagnetic coupling. The entanglement also modifies the matching condition introducing a dependence of the optimal value of load impedance on the line length.

Index Terms — Electromagnetic compatibility, kinetic inductance, nano-circuits, nano-electromagnetism, quantum devices, quantum entanglement.

I. INTRODUCTION

TODAY’s achievements of nanoelectronics allow utilization and manipulation of small collections of atoms and molecules, such as semiconductor heterostructures, quantum wells, quantum wires and quantum dots [1-3], different forms of nanocarbon (spherical fullerenes, graphene, carbon nanotubes [4-7]), noble metal nanowires [8], organic macromolecules and organic polymers [9]. The increasingly intensive penetration of nanotechnologies lead to the birth of the so-called “nanoelectromagnetism” [10-11], a novel branch of applied science related to the interaction of electromagnetic radiation with quantum mechanical low-dimensional systems.

One of the crucial aspects of electromagnetism for electronic devices and systems is related to their electromagnetic compatibility, i.e., of their ability to operate successfully, with controlled levels of emissions and with a suitable degree of robustness to unwanted electromagnetic couplings via various mechanisms of interference [12-14].

However, with the transition of electronics to nanoscale new physical phenomena as well as new materials’ properties need to be studied. Quantum effects, such as: discrete energy spectrum of charge carriers, existence of phonons, ballistic transport and tunneling, many-body correlations, interface effects, and so on, manifest themselves jointly with classical electromagnetic interactions [15-16]. As a result, the classical design based on the phenomenological separate analysis of physical properties of electric circuit elements and functional properties of devices and systems becomes invalid with respect to nanoelectronics. These considerations lead to the conclusion that the “classical” EMC, completely based on macroscopic electrodynamics, must be deeply revised starting from the basic concepts and opening the era of “nanoEMC” [17-19].

For instance, the classical scaling rules used to design integrated circuits (ICs) and to implement EMC solutions are based on the macroscopic behavior of the electrical parameters, such as inductances and capacitances. However, the quantum terms appearing in the models of nanoelectronics change the dependence of such electrical parameters on frequency, geometry and temperature. Consequently, the classical EMC concepts like coupling, shielding, and matching, should be reconsidered, along with the classical solutions to such issues. In other words, nanoEMC poses new challenges in modeling devices and systems, in establishing new design rules and in assessing reliable characterization procedures.
NanoEMC modeling assumes the self-consistent solution of Maxwell’s equations with the quantum transport equations for charge carriers. Transition from the macroscopic to the atomic scale can be performed via either of the three distinct approaches to electron transport modeling: classical, semi-classical, and quantum. Many works were devoted to this topic, investigating nanomaterials for EMC applications like shielding [20-21], modeling the interactions between EM fields and nanostructures [22-23], and modeling nano-interconnects [24-29].

The classical approach, with the most limited validity area, is the simplest one. For example, Drude model of conductivity in metals [16], considers the conducting electron as a classical particle, which moves in the electric field while encountering inelastic collisions with a randomly vibrating ion lattice.

The semi-classical approach is based on the concept of a particle ensemble behaving as a non-ideal gas with quantum effects taken into account by replacing the real particle mass by the corresponding effective value [15-16]. In these models, electrons are unable to tunnel through barriers. In collision events, the electrons’ scattering is inelastic, thus the kinetic energy of incident particles is not conserved. The transmission line (TL) model for carbon nanotube (CNT) interconnects presented in [28-29] may be noted as an example of such a semi-classical approach.

On the contrary, at the molecular or atomic scale a quantum approach is needed, since the transport is governed by the wave-like behavior of the electrons. Thus, unwanted interactions between elements exist both due to electromagnetic coupling and quantum phenomena, such as tunneling, spin-orbit interaction, and various many-body effects including dipole-dipole and spin-spin interactions. As a result, appears the entanglement of quantum states, which produces the long-living and long-distance correlations of quantum origin in electric circuits [16]. For their description the complete Maxwell-Schrödinger model is necessary. One of the most convenient forms of it is based on the concept of generalized susceptibilities (Kubo-approach) [16].

This paper discusses the concepts of coupling and matching in a nanoscale signaling system composed of nanowires connecting quantum devices in the entangled states. The interest in this type of systems from the EMC point of view stems mainly from their potential as digital elements for quantum computing and quantum informatics [1]. In particular, the paper compares predictions of the conventional EMC-theory with those of nanoEMC taking into account the coupling of quantum nature, due to the entanglement.

The paper is organized as follows. Section II is devoted to the modeling. First, the TL model for nano-interconnects based on metal nanowires (NWs) or carbon nanotubes (CNTs) is briefly recalled. Then, an equivalent lumped model to describe a pair of quantum devices in entangled state is proposed. Section III deals with the analysis of two complementary case-studies: the first one refers to a nanointerconnect with a crosstalk noise induced by an unwanted electromagnetic coupling. The second proposes two uncoupled nanowires terminated on two quantum dots in the entangled state. The summary and conclusive remarks are given in Section IV.

II. MODELING A NANOSCALE SIGNALING SYSTEM

In this paper we analyze the behavior of signaling nanocircuits composed of quantum devices connected by nanowires, which play a role of transmission lines. Section II.A summarizes the quasi-classical model of transport in nanowires made of metals or carbon nanotubes, presented in [28-29] and [32], while Section II.B proposes an equivalent lumped model of the quantum devices in the entangled state, in the framework of the circuit theory.

A. Modeling nanoscale interconnects

The considered nano-interconnect (Fig.1) is based on the two signal lines suspended in air above a perfectly conducting ground. The interconnect length is assumed infinite in the z-direction. This model adequately describes a real interconnect if the diameters of the signal lines are much smaller than their lengths (typically, at least an order of magnitude). The conducting material for the signal lines may either be a metallic NW or a CNT. The wire diameter is assumed to be large enough (at least 1 nm) for having a local crystal structure of the wire, which allows the using of semi-classical transport model. For assumed ratio between diameter and length, the conducting nanowire can be regarded as a one-dimensional (1-D) material, with the charge transport characterized by two quantum-confined directions and single one unconfined.

The operating frequencies range is from zero up to some THz, so that:

(i) the cross section diameter \( D \) of both nanowires is electrically small;
(ii) the manifestations of transverse currents in the nanowires may be neglected;
(iii) only the intraband transitions in the particle movement are taken into account.

Since the conducting electrons are laterally quantum confined, they occupy the narrow energy subbands, instead of the ordinary wide bands found in the bulk materials. Along the longitudinal axis, the lattice exhibits translational symmetry and is long enough to consider the longitudinal electron wave-number as a continuous value. Such type of energy spectrum may be evaluated by means of the tight-binding approximation (e.g., CNTs, [30]), or first principle calculations (e.g., copper NWs, [31]).

In the semi-classical approximation, the electrons are considered as a 1-D gas described by the Boltzmann transport equation. The quantum nature of the conductive electrons is accounted via a non-parabolic dispersive law [28-29].

![Figure 1. Reference geometry for the nano-interconnect: to signal lines above a Perfect Electric Conducting (PEC) ground.](image-url)
In the momentum domain the traveling wave can be presented as $\exp[i(\omega t - k_0z)]$, where $\omega$ is the radian frequency and $\beta$ is the wavenumber. By solving the transport equation in such a domain, we get the following expression for the longitudinal component of the current density, $J_z$:

$$
\tilde{J}_z(\beta, \omega) = \tilde{\sigma}_z(\beta, \omega) \tilde{E}_z(\beta, \omega),
$$

where $\tilde{E}_z$ is the longitudinal component of the electric field, and $\tilde{\sigma}_z$ is the longitudinal conductivity. The latter can be evaluated as the sum of all the contributions of the subbands:

$$
\tilde{\sigma}_z(\beta, \omega) = \sum_{\mu=1}^{N} \tilde{\sigma}_\mu.
$$

Here the number $N$ refers to the subbands that significantly contribute to the conduction, namely those for which the energy gap with respect to the Fermi level is small enough, usually $|E_{\mu} - E_F| \leq 5k_B T$, where $E_F$ and $E_{\mu}$ are, respectively, the energies of the Fermi level and of the $\mu$-th energy subband (in eV), $T$ is the absolute temperature and $k_B$ the Boltzmann constant (in eV/K). Assuming the collision frequency $\nu$ to be constant for all the subbands close to the Fermi level, it results:

$$
\tilde{\sigma}_\mu(\beta, \omega) = \frac{2e^2 v_F}{\pi \hbar} \frac{1}{\omega - i\nu} M \left[ 1 - \xi(\omega) \left( \frac{v_F \beta}{\omega - i\nu} \right)^2 \right],
$$

where $\hbar$ is the reduced Planck constant, $v_F$ is the Fermi velocity, $M$ is the equivalent number of conducting channels [28], and the quantities $X$ and $\xi(\omega)$ depend on the material as signal trace. For the case of CNT, the quantity $X$ is its circumference, and [32]:

$$
X = \pi D, \quad \xi(\omega) = 1,
$$

while for a nanowire, $X$ is its cross section $S_w$, and [33]:

$$
X = S_w = \frac{\pi}{4} D^2, \quad \xi(\omega) = \frac{1}{3} \frac{1 + 1.8i\omega / \nu}{1 + i\omega / \nu}.
$$

By combining (3) and (1), we get a generalized Ohm’s law:

$$
\left[ 1 - \psi(\omega) \beta^2 \right] \tilde{J}_z(\beta, \omega) = \frac{\sigma_0}{1 + i\omega / \nu} \tilde{E}_z(\beta, \omega),
$$

where

$$
\psi(\omega) = \frac{\xi(\omega) v_F^2}{v^2 (1 + i\omega / \nu)^2},
$$

$$
\sigma_0 = \frac{2v_F M}{\nu R_0 X}.
$$

and $R_0 = \hbar / e^2 = 12.9 \text{k}\Omega$ is the quantum resistance. Note that the number of conducting channels $M$ strongly depends on the chirality, size and temperature [34]-[36].

If we assume an uniform distribution of the current, i.e., $I(z, \omega) = J(z, \omega) X$, we can multiply both parts of (6) by $X$ and rewrite it in spatial-frequency domain (using $\beta = -\partial^2 / \partial z^2$):

$$
I(z, \omega) + \psi(\omega) \frac{\partial^2 I(z, \omega)}{\partial z^2} = \frac{\sigma_0 X}{1 + i\omega / \nu} E_z(z, \omega).
$$

The second term on the left hand side introduces a spatial and frequency dispersion, whereas the coefficient of the electric field introduces a frequency dispersion.

We assume the electromagnetic field to be low enough for using the linear approximation with respect to it (namely for voltage values, $V < k_B T / \epsilon$). Thus, it is possible to derive a simple linear TL model for the nano-interconnect schemes in Figure 1 (e.g., [28], [29], and [32]) by coupling (9) with Maxwell’s equations. Assuming a single line (one wire above the ground), we would have the TL equations:

$$
- \frac{dV}{dz} = (R_{TL} + i\omega L_{TL}) I, \quad - \frac{dI}{dz} = i\omega C_{TL} V,
$$

where the per-unit-length (p.u.l.) resistance, inductance and capacitance would be given by:

$$
R_{TL} = \frac{\nu v_F}{\Theta(\omega)}, \quad L_{TL} = \frac{L_a}{1 + \frac{\nu v_F}{\Theta(\omega)}}, \quad C_{TL} = C_E, \quad\text{with}
$$

$$
\Theta(\omega) = 1 + \frac{C_E}{C_q} \frac{\xi(\omega)}{\nu v_F / \omega},
$$

$$
L_a = \frac{1}{\nu \sigma_0 X} = \frac{R_0}{2v_F M}, \quad C_q = \frac{1}{L_a v_F} = \frac{2M}{R_0 v_F}.
$$

In (11)-(13) appear two novel terms with respect to the ordinary macroscopic TL-equations: the p.u.l. kinetic inductance $L_a$, related to the mass inertia of the conduction electrons, and the p.u.l. quantum capacitance $C_q$, related to the quantum pressure arising from the zero-point energy of such electrons.

In spite of their different physical meaning, the generalized TL model with parameters (11)-(12), to be valid for CNT lines and metallic NWs, is consistent with the classical TL model for a macroscopic line. To show it, we assume the simple case of a copper wire above the ground, with $D = 400 \text{nm}$, and $H=2D$ (see Fig.1). For copper NWs the number of channels $M$ at room temperature may be calculated by $M = a S_w + b$, where $a = 0.11 \text{nm}^{-2}$, and $b = 3.036$ (see [31], eq.(4)). In this case it is $M \approx 1.4 \times 10^4$. For such a case, it is $\Theta \approx 1$ and $L_a \ll L_T$ , and so (11), (12) yield the classical TL parameters: $R_{TL} = 1/(\sigma_0 S_w)$, $L_{TL} = L_M$, $C_{TL} = C_E$.

Here we meet a important difference between macroscopic and nanoscale TLs. For macroscopic TLs the working mode is the Transverse ElectroMagnetic (TEM) wave, with propagation velocity $c = 1/\sqrt{LC}$. The working mode in nano-TLs is a surface wave with rather large longitudinal component, thus from (13) we obtain $v_F = 1/\sqrt{L_a C_q}$. Such coupling condition between linear electric parameters should be taken into account in EMC-applications at nanoscale.
A dramatic improvement of device integration and computational information transport, storage and processing promises a new paradigm for quantum computing. As shortly mentioned in the introduction, candidates for basic digital elements of quantum origin computing is still far from being defined and assessed, the recent monochromatic with frequency approximately equal to the transition frequency. In addition, we assume that the dimension of the confinement size $2a$ is electrically small at the given frequency, thus the single quantum device may be modeled as a one-port lumped element. We indicate with $V(t)$ and $I(t)$ the voltage and the current intensity of the one-port element. Following [42-43], the device equivalent impedance may be expressed in term of the quantum polarizability, $\alpha(\omega)$, as:

$$Z_L(\omega) = \frac{V}{I} = \frac{i}{\alpha(\omega)\omega} A_{\text{eff}},$$

(14)

where, following the Kubo approach [44], it is:

$$\alpha(\omega) = \frac{1}{\hbar} \frac{2\mu^2 \omega_b}{\omega^2 - \omega_0^2 + i\gamma^2}$$

(15)

In (14)-(15), $\omega_b$ is the frequency of quantum transition between the two stationary states, $\mu$ is the dipole moment of the transition, and $\gamma$ is the spontaneous emission decay.

One of the possible quantum coupling mechanism between the two lumped devices, for instance, exists via dipole-dipole interactions (another coupling mechanism is spin-spin interaction, etc.) [45]. As a consequence, the original two-level energy spectra of the uncoupled quantum devices are transformed now into a one-piece four-level one. In this case, two intermediate levels correspond to the so-called entangled states [39]: although only one quantum device is excited, this excitation is distributed with the same probability between both devices, due to quantum correlations [39]. It is necessary to distinguish between symmetric (superradiant) and anti-symmetric (subradiant) states, which are characterized by different values of transition energies and emission decays [39].

The characteristic of the two-ports representing the pair of quantum devices in the entangled states may be again found by using the Kubo approach [44]. For the symmetric state we have

$$I(\omega) = Y(\omega) V(\omega) = \frac{1}{2Z_L(\omega)} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} V(\omega).$$

(16)

where $V = [V_1, V_2]^T$ and $I = [I_1, I_2]^T$ are the voltages and currents at the two ports, whereas $Y(\omega)$ is the matrix of equivalent conductivity, and $Z_L$ is given by (14).

III. COUPLING AND MATCHING AT NANOSCALE

In this Section we discuss the concepts of coupling and matching, referring to the simple two-channel signaling system in Fig.3. In order to study the coupling, a typical condition is given by switching on only one driver (for instance which connected to line 1) and evaluating the induced voltages at the near and far end of the other line (line 2), usually normalized to the driver voltage. As for the matching, we will discuss the behavior of the reflection coefficient at the load section.

Here we discuss two complementary cases: (i) two electromagnetically coupled nanowires terminated with two classical uncoupled loads; (ii) two uncoupled ideal transmission lines with quantum devices in entangled states.
A. Electromagnetically coupled nano-interconnects

In this subsection we consider for crosstalk analysis [46] the MTL in Fig. 1, used as a two-channel line in the circuit shown in Fig. 4. The drivers and receivers are modeled as real voltage sources and capacitors, respectively.

In macro-scale modeling, the crosstalk voltages at the near and far ends of electrically short lines are simply given by [46]:

\[
\frac{V_{NE}(\omega)}{U_1(\omega)} = i\omega(K^\cap C_m + K^\ind L_m), \quad (17)
\]

\[
\frac{V_{FE}(\omega)}{U_1(\omega)} = i\omega(K^\cap C_m - K^\ind L_m), \quad (18)
\]

where \(l\) is the line length, \(C_m\) and \(L_m\) are the p.u.l. mutual capacitance and inductance of the line, and \(K\) are coupling coefficients related to the terminal impedances. The subscripts \(NE, FE\) relate to near-end and far-end voltages, whereas the superscripts \(\cap, \ind\) correspond to capacitance and inductance.

In the wide-separation approximation, the self and mutual terms of the inductance matrix \(L\) for the MTL in Fig. 1 are [46]:

\[
L_s = \frac{\mu}{2\pi} \ln \left( \frac{2H}{D/2} \right), \quad L_m = \frac{\mu}{4\pi} \ln \left( 1 + \frac{4H^2}{S^2} \right) \quad (19)
\]

whereas the p.u.l. capacitance may be evaluated by \(C = \mu e L^{-1}\).

We start from a given set of values for the geometrical parameters \(D, H, S,\) and \(l\), that provide crosstalk voltages (17) and (18) satisfactorily low for the considered frequencies and loads. In classical applications, this happens, for instance, by following the so-called “3W spacing rule”, i.e. by assuming the edge-to-edge distance between the signal traces (in this case, \(S\)) to be equal to 3 times the trace width \(W\) (here replaced by the diameter \(D\)). This is a common compromise between acceptable routing density and crosstalk [14]. We can define a scaling rule reducing the dimensions by the same factor \(x\):

\[
D' = Dx, \; S' = Sx, \; H' = Hx. \quad (20)
\]

After the scaling (20), the crosstalk noise is left unvaried, since the matrices \(C\) and \(L\) do not change, according to (19). If we scale down also the line length \(l\), than the crosstalk noise is proportionally reduced. This simple scaling rule may be rigorously used for lossless lines, and is approximately true for lossy lines too, if the internal inductance can be neglected.

The same behavior may be still found after removing the hypothesis of electrically short lines, hence evaluating the noise voltages as solution of a TL problem. Figure 5 shows the amplitude of the far-end crosstalk voltage, obtained for a classical TL model (pair of copper wires), assuming for parameters defined in Fig. 1 the values \(D=0.4\) mm, \(H=D, S=3D\), and \(l=10\) mm. For the loads shown in Fig. 4 we take \(C_L = C_D = 1\) pF and \(R_D = 80\) Ohm. The result obtained by using (20) with a factor \(x = 10\) is the same, since the two curves coincide. The crosstalk noise shows a peak dependent on the value \(C_L\), and shifts to higher frequencies as \(C_L\) decreases. In addition, the solution exhibits the typical TL resonances for \(\beta R = n\pi\). The line length shortening reduces the level of crosstalk and shifts mentioned resonances to higher frequencies, enlarging the frequency range with negligible level of noise.

We now study the same problem for the nanoscale TL model (interconnect based on two CNTs). Geometry of the problem again corresponds to Fig. 1. We consider either multi-wall CNT of diameter \(D = 20\) nm, and metallic single-wall CNT, with \(D = 2\) nm. For other parameters, the chosen values are \(H = D, S = 3D, l=0.1\) mm. The interconnect is used in a circuit like that in Fig. 4, with \(C_L = C_D = 0.1\) pF and \(R_D = 3\) kOhm. To take into account the contact resistance, an additional lumped resistor of 0.5 Ohm is added at each termination. The far-end crosstalk noise is reported in Fig. 6, considering again the cases when \(C_L\) or \(D\) are reduced by a factor of 10.
As observed for the classical model, the reduction of the load capacitor shifts the peak of the noise to higher frequency values. The novelty is the sensitivity to the wire diameter that breaks the scaling rule observed in the classical solution. The reason is due to the impact of the number of conducting breaks the scaling rule observed in the classical solution. The values. The novelty is the sensitivity to the wire diameter that load capacitor shifts the peak of the noise to higher frequency values. As pointed out in subsection II.A, this happens for the classical model, the reduction of the noise to higher frequency values.

B. A pair of quantum devices in entangled states

We now discuss the case of two uncoupled lossless TLs, fed by two ideal voltage sources and terminated with a pair of quantum devices in entangled states, as schematically depicted in Fig.7. The lines are assumed to be identical, thus having the same wavenumber $\beta = \omega \sqrt{LC}$ and characteristic impedance $Z_C = \sqrt{L/C}$. The device is the pair of quantum dots (see Fig.2) assumed to be in the superradiant entangled state, so that (16) holds. As shown in Fig.7, the device may be represented by a simple $\pi$-type two-port, derived from (16), with the self and mutual admittance given by:

$$Y_s = Z_L^{-1}, \quad Y_m = -(2Z_L)^{-1}.$$  

Note that the coupling element, $Y_m$, has a negative real part, without contradicting the thermodynamic equilibrium. As noted by Schrödinger [48], “The whole system can be less uncertain than either of its entangled parts”. It means that the whole equivalent circuit is better specified than its elements. In our case, a negative resistance of circuit element means that there is a special type of energy transfer inside the system, different from the outside energy supply. It strongly contradicts the intuitive concepts of classic crosstalk, in which the electromagnetic coupling should be described only by passive elements [12-14], [46]. For these reasons, the considered system is not equal to a pair of coupled harmonic oscillators and has no analogs in classical electrodynamics.

To study the circuit in Fig.7, each line may be represented as a two-port network via the transmission matrix [46]:

$$T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}, \quad T_{12} = -iZ_c \sin(\beta), \quad T_{21} = -iZ_c^{-1} \sin(\beta).$$  

As in Section III.A, we evaluate the coupling by studying the voltage on the load $Z_L$ of line 2, when $U_1 = 1V$ and $U_2 = 0V$. By coupling these boundary conditions to line matrix (21), we obtain a simple expression for the far-end voltage:

$$V_{FE} = \frac{1}{2} T_{12} (T_{12} - Z_L T_{21}) = \frac{1}{2} \frac{\tan(\beta)}{\sin(\beta) - iZ_L/Z_C} \sin(\beta).$$  

In the following we assume that each quantum device is made by the GaAs double quantum dot proposed in [49] as a qubit for quantum computing. We consider the following values for the parameters in (14)-(16): $a \approx 20$ nm, $\omega_0 = [1 \pm 1.5] \cdot 10^{15}$ rad/s, $\gamma = 3 \cdot 10^7$ Hz and $\mu = 7.59$ $\mu$. The frequency behavior of the far-end voltage is plotted in Fig.8, with varying values of the frequency of quantum transition $\omega_0$. As a consequence of the entanglement, the active line is able to excite signals into the victim line even in absence of inter-line coupling. The spectrum plotted in Fig.8 shows a resonance for $\omega = \omega_0$, besides the classical resonances due to the transmission lines (for $B = n\pi/2$).

$$U_1 \quad \text{Line 1} \quad Y_m \quad \text{YDs} \quad Y_s \quad \text{Line 2} \quad U_2$$

Figure 7. Equivalent scheme for the two uncoupled lines terminated on two coupled quantum devices, represented as a $\pi$-type two-port. The admittances are $Y_s = Z_L^{-1}, \quad Y_m = -(2Z_L)^{-1}$, where $Z_L$ is given by (14).
C. Matching conditions

In this subsection we discuss the concept of matching, referring to a single transmission line made by one of the signal wires in Fig.1 and the ground plane. In the classical TL theory, the amplitude of the reflection coefficient at the load section is given by [46]:

\[ \Gamma_L = \frac{1 - \zeta}{1 + \zeta}, \quad \zeta = \frac{Z_C}{Z_L} \]  

which leads to the classical matching condition

\[ Z_L = Z_C, \]  

giving \( \Gamma_L = 0 \). As it is well known, for a lossless line the characteristic impedance becomes a pure resistance \( Z_C = Z_{C0} = \sqrt{L/C} \), therefore a resistive load \( Z_L = Z_{C0} \) would give a perfect matching at any frequency. For a lossy line the characteristic impedance become complex and frequency dependent: for instance, for the simple case of a lossy RLC line without skin-effect, it is:

\[ Z_C(\omega) = \frac{R + i\omega L}{i\omega C}. \]  

It means that the load \( Z_L = Z_{C0} \) would give a perfect matching condition only in the high frequency limit \( (\omega \rightarrow \infty) \).

If we consider the nanoscale TL, the situation qualitatively changes: the frequency range of satisfactory matching may be enlarged as a consequence of the presence of the kinetic inductance \( L_k \). Let us consider, for instance, a metallic single-wall CNT with \( D = 2nm, \quad H = D, \quad \text{and} \quad l = 1 \text{mm} \). In this case, \( L_k/L_{at} = 1.3 \times 10^4 \) and \( C_E/C_q < 1 \). Figure 9 plots the load reflection coefficient (24), evaluated with and without the contribution of \( L_k \). If, for instance, in a given problem a satisfactory matching can be considered when the reflection coefficient less or equal to 0.1, then from Fig.9 is evident that in presence of kinetic inductance enlarges the region where this condition is fulfilled.

As shown for the crosstalk analysis, this effect vanishes when the number of channels \( M \) is large enough to lower the value of the kinetic term (for large diameter NWs or large CNT bundles).

Let us now discuss the concept of matching when an ideal TL is ended with a pair of quantum devices in the entangled state (see Fig.7). Assuming one of the two lines to be inactive, the reflection coefficient at the load of the other line may be expressed as:

\[ \Gamma_L = e^{i\beta l} \frac{2e^{-i\beta l} \cos(\beta l) - \zeta}{2e^{i\beta l} \cos(\beta l) + \zeta}, \quad \zeta = \frac{Z_C}{Z_L}, \]  

which provides a new matching condition, that redefines (25):

\[ Z_L = \frac{Z_C}{2} (1 + i\tan(\beta l)), \]  

If we would impose the classical matching condition (25), we would obtain an energy reflection coefficient \( [\Gamma_L]^2 \in [0.1, 1.0] \), depending on the phase shift \( \beta l \). Condition (28) imposes an optimal phase shift between the source and the load in order to obtain total absorption, whereby the matching impedance becomes complex and depends on the line length. Thus, we obtain another result, important from EMC-point of view: quantum entanglement is able to break down the regime of matching between transmission line and the load.

IV. CONCLUSIONS

The new phenomena introduced by quantum effects in nanoelectronic systems suggest a deep revision of the concepts adopted in the classical EMC analysis. The phenomenological approaches based on separable description of physical properties...
of materials and electromagnetic response of electronic devices and systems become invalid on nanoscale. Instead, a self-consistent modeling of dynamics of quantum charge carriers and classical electromagnetic fields becomes necessary. The most efficient approach to this problem is based on the fundamental physical theory of generalized susceptibilities (Kubo-approach).

In this paper we have analyzed two case-studies: coupled nanoscale interconnects terminated with uncoupled loads, and coupled quantum devices in the entangled state connected to two uncoupled interconnects. In both cases, the coupling and matching conditions are dramatically modified, as compared to the classical EMC theory.

As for the nano-interconnects, their equivalent electrical parameters are affected by kinetic inductance and quantum capacitance, which introduce a new behavior with respect to the dimension scaling. The classical scaling rules no longer hold, as well as the separation rules adopted to mitigate the crosstalk. The kinetic inductance, however, plays a beneficial role, enlarging the attainable frequency range of the satisfactory matching of the load with the line. A pair of quantum loads coupled via entanglement have been considered. As a result of quantum coupling, a voltage arises in the passive line, in spite of its electromagnetic decoupling with the active one. Thus, a novel concept arises: since the entanglement is a “spooky action at distance” his effects may surpass those due to the near field electromagnetic coupling, which are modeled by the cross-admittances. Quantum entanglement introduces a novel matching condition between the line and the load, with the optimal value of the load impedance dependent on the line length.

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