The creation of supra-thermal densities of high energy phonons in He II

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Abstract

We report the new phenomenon that high energy phonons can be created from low energy phonons. This arises because the dynamics of phonons in propagating pulses are quite different to those in isotropic phonon distributions. A pulse of low energy phonons rapidly thermalises by three phonon processes. On a much longer time scale four phonon processes occur within this phonon cloud which create high energy (10K) phonons that cannot spontaneously decay. These phonons have a lower velocity and so are lost from the back of phonon cloud; their deficit is restored continuously by four phonon processes. These now isolated high energy phonons are very stable and propagate ballistically behind the low energy phonons, so giving the two pulses which are detected in experiments. For long pulses the high energy phonons may also decay within the cloud, however the available low energy phonons for scattering are confined to a narrow-angle cone, so the decay probability is very low because the four phonon process requires larger angle scattering. A supra-thermal density of these high energy phonons is predicted.

Keywords: phonon pulses; liquid helium; phonon interactions; supra-thermal phonon density

1. Introduction

It has recently become apparent that the dynamics of phonons in pulses are quite different to those in isotropic distributions. This arises because the restricted momentum directions in a pulse coupled with strongly angular dependent phonon-phonon interactions, forbid many scattering processes that can occur in an isotropic distribution. This has dramatic consequences for the lifetimes of phonons especially those at high energy, and also on the dynamic equilibrium populations of these phonons, indeed supra-thermal populations are predicted.

This subject originated from the observation of a small discrepancy in the time of flight of high energy, h, phonons propagating in cold liquid ^{4}He. They appeared to be going too fast. It was conjectured and later verified that this was due to the h phonons being created from low energy, l, phonons at some distance in front of the pulsed heater. The fast signal is then simply due to the higher velocity of the l phonons compared to the h phonons. The creation process involves both three and four phonon processes (3pp and 4pp respectively). The

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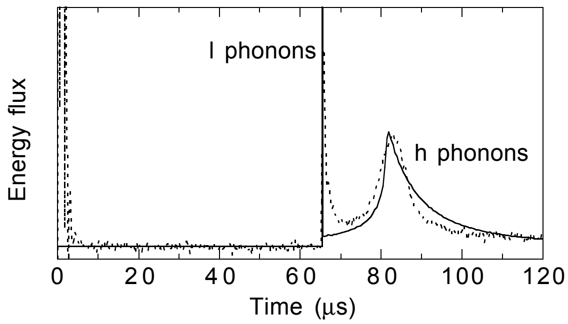

Fig. 1. l and h phonon signals from a single input pulse after propagating 15.6mm in liquid helium, broken line: measured signal; and solid line: calculated.

model for the creation process depends on the time constants for the 3pp and 4pp creation and decay rates. This theory gives a good description of the experimental results and makes a number of predictions. We will describe the various stages of its development.

High energy phonons are those with energy $\varepsilon > \varepsilon_c$ where ε_c is the energy where $\varepsilon(p) = cp$. These phonons are stable against spontaneous decay, as energy and momentum must be conserved, and in cold helium have lifetimes which tend to infinity as $T \to 0$. They were first studied by quantum evaporation as $\varepsilon_c = 10$K is greater than the binding energy of atoms in liquid ^{4}He of 7.16K. This allows phonons to evaporate atoms in a one to one process. The time of flight of these phonons was found to be typically $5\mu s$ shorter than that expected from the group velocity derived from the dispersion curve determined by neutron scattering and a path length measured from the heater surface, as it was then thought that the detected h phonons were injected by the heater [1].

This small discrepancy remained unexplained until it was realised that this idea should be abandoned and replaced with the notion that h phonons are created in the helium, up to a few millimetres in front of the heater, [2] and [3]. *Prima facie* this might be considered a strange conjecture as it involves high energy phonons being created from low energy phonons. This requires an increase of a factor of ∼5 in the average phonon energy, $\bar{\varepsilon}$, as the l and h phonons have $\bar{\varepsilon}_l \sim 2$K and $\bar{\varepsilon}_h \sim 10$K respectively.

Conclusive proof that most of the h phonons are created in the liquid helium and not injected by the heater came from measurements at pressure, [2] and [3]. For this the h phonons had to be detected in the liquid. Fig. 1 shows the l and h phonon signals created by a single short heater pulse; the distance between the heater and the bolometer is 15.6mm. As pressure is increased so $\varepsilon_c(p)$ decreases and it is clear that the h phonons are in a narrow energy range just above $\varepsilon_c(p)$ which is to be expected from the scattering processes in the helium: the exponential decrease of the l phonon density with ε, at high ε, makes it most probable that phonons just above $\varepsilon_c(p)$ are created. Recent experiments have shown that the probability of creating h phonons decreases with distance away from the heater [4].

Another germane experimental finding is that the generation of h phonons requires relatively high heater powers and short pulse lengths, t_p, compared to generating R$^+$rotons [1]. The amplitude of the h phonon signal initially increases with pulse lenth but then saturates at $\sim 10^{-7}$s. Again this would not occur if the h phonons were injected by the heater and it is a characteristic behaviour that must be explained theoretically.

2. Qualitative Description of the Model

The heater clearly injects l phonons into the liquid helium. There are two channels for emission, the peak and background channels [5]. Phonons in the peak are elastically transmitted through the solid-liquid interface and so are confined to a narrow angle cone of directions in the helium. It is found that even for polycrystalline gold films on glass substrates that the l phonons are peaked in the direction normal to the substrate [6] and [7]; i.e. they are confined to a narrow cone in momentum space as shown in Fig. 2. The high phonon density in the peak means that these l phonons thermalise to a substantial temperature of about 1K, [8] and [9]. We shall see that this high temperature is crucial for the production of h phonons.

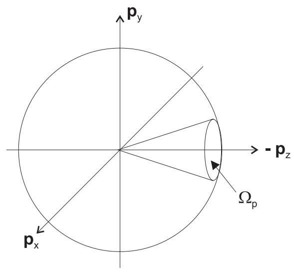

Fig. 2. The occupied cone in momentum space.

The l phonons scatter by the three phonon process (3pp). These interactions cause thermalisation in $\sim 10^{-10}$s [9] which is very fast compared to all the other times in the system. This means that, even when the energy of the phonon cloud changes, we can always treat the l phonons as an equilibrium system with an effective temperature, T_{e}, which includes all phonons with energy $\varepsilon < \varepsilon_c$. The l phonon cloud is spatially well defined in the liquid helium and maintains a constant length, $L = ct_p$ where t_p is the pulse period, as it propagates.

The h phonons are created within the l phonon cloud. They cannot be created by 3pp but can be by the much weaker 4pp [2]. We envisage two l phonons with for example combined energy around 12K, interacting and annihilating and creating one h phonon and a low energy phonon with energies near 10K and 2K respectively. The h phonon may also decay by the reverse process. The h phonons are essentially created in the direction of the average momentum of the l phonons.

We can represent the l and h phonons as two weakly interacting systems, as shown in the Fig. 3, with an interchange between the two systems by the relatively slow (10^{-7}s) 4pp [11]. There is also loss by dispersion as the h phonons have a lower group velocity than the l phonons, 190ms^{-1} and 238ms^{-1}respectively, so they can be left behind the moving l cloud. The deficit of h phonons in the l cloud is restored by further 4pp creation processes. The loss of h phonons from the back of the cloud represents an energy loss and causes the cloud to cool.

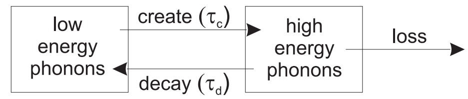

Fig. 3. The l and h phonon systems interact via the 4pp and h phonons are lost by dispersion.

This process is shown schematically in Fig. 4. As the cloud cools so the creation rate of h phonons drops rapidly. So for short pulses, at only a few millimetres in front of the heater the h and l phonon pulses become spatially separated and stable. They then propagate ballistically and give two pulses at the detector.

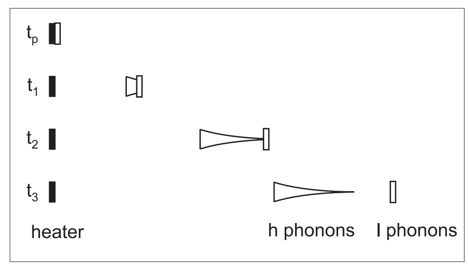

Fig. 4. A schematic diagram of the spatial disposition of h and l clouds at four sequential times.

For longer pulses the dispersive loss becomes less important than the decay processes because of the greater time needed for an h phonon to escape. In the limit of no dispersive loss the h and l phonons will eventually come to a dynamic equilibrium which is determined by the two time constants τ_c and τ_d for the creation and decay of h phonons respectively.

These time constants are affected by the restricted range of angles in the momentum cone. The decay processes are most changed from that in an isotropic phonon distribution. This can be seen in the example shown in Fig. 5. The h phonon can interact with l phonons with angles in the shaded sector. The l phonons in the cone lie outside this sector so in this example the h phonon cannot decay. This means that τ_d is much longer in the pulse than in an isotropic phonon distribution.

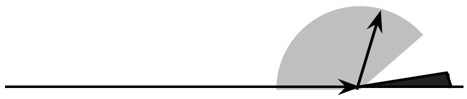

Fig. 5. The momenta of h and l phonons. The h phonon can only scatter from low energy phonons with angles in the light shaded sector but the l phonons are only in the dark shaded cone, $\varepsilon_h = 10.2$K and $\varepsilon_l = 2$K.

3. Analytical model

The kinetic equation for the distribution function, $n_h(\varepsilon)$, of the h phonons of energy (ε) in the moving frame of the l phonons is [10]

$$\frac{\partial n_h}{\partial t} + \overrightarrow{u_h} \cdot \nabla n_h = \frac{n_h^{(0)}}{t_c} - \frac{n_h}{t_d} \qquad (1)$$

where $n_h^{(0)}(\varepsilon)$ is the equilibrium Bose-Einstein distribution function for h phonons at temperature T, $\overrightarrow{u_h} = \overrightarrow{c_h} - \overrightarrow{c}$ is the relative velocity and $\overrightarrow{c_h}$ and $\overrightarrow{c}$ are the velocities of the h and l phonons respectively, t_c is the four phonon scattering time for the creation of h phonons in an equilibrium distribution of l phonons in a momentum cone at temperature T, and t_d is the corresponding decay time.

Multiplying equation (1) by ε and integrating over the phase space occupied by the h phonons we get the equation for the energy density, E_h, for the h phonons

$$\frac{\partial E_h}{\partial t} + u_h \frac{\partial E_h}{\partial z} = \frac{E_h^{(0)}}{\tau_c} - \frac{E_h}{\tau_d} \qquad (2)$$

where we have made a one dimensional approximation with the z axis taken to be antiparallel to the propagation direction, the energy of the equilibrium density of h phonons, $E_h^{(0)}$, in the high energy tail, $\varepsilon \geq \varepsilon_c$, of the Bose-Einstein distribution is given by

$$E_h^{(0)} = \frac{\Omega_p}{(2\pi c)^3} T \varepsilon_c^3 \,\mathrm{e}^{-\varepsilon_c/T} \qquad (3)$$

where Ω_p is the occupied solid angle in momentum space (see Fig. 2), τ_c is the four phonon creation time and τ_d is the four phonon decay time in the pulse. We can use energy independent times as the h phonon distribution is strongly peaked just above ε_c. The creation and decay times are not the same as they would be in a spherically symmetric distribution of phonons because of the restricted momentum cone in the pulse, in general they are both longer than τ_4, the creation and decay time in an equilibrium spherical distribution, but $\tau_d >> \tau_4$.

The energy for the creation of h phonons comes from the l phonons, so from equation (2) we get the rate of change of the energy density of the l phonons :

$$\frac{\partial E_l^{(0)}}{\partial t} = -\frac{E_h^{(0)}}{\tau_c} + \frac{E_h}{\tau_d} \qquad (4)$$

where $E_l^{(0)}$ is the energy density of l phonons given by

$$E_l^{(0)} = \frac{\Omega_p \pi}{120 c^3} T^4 \qquad (5)$$

From equations (4) and (5) we obtain an equation for the cooling rate of the l phonons in the pulse:

$$\frac{\partial T}{\partial t} = -\frac{T}{4E_l^{(0)}} \left(\frac{E_h^{(0)}}{\tau_c} - \frac{E_h}{\tau_d} \right) \qquad (6)$$

The boundary condition is $E_h(z = 0, t) = 0$ as the h phonons move from the front of the l pulse, $z = 0$, to the back , $z = L$, and we take the initial conditions to be $T(t = 0) = T_0$= constant and $E_h(z, t = 0) = 0$.

3.1. Short pulses

If the pulse is short enough then all the h phonons that are created are lost from the back of the l pulse, then the term E_h/τ_d in equation (6) can be neglected. This occurs when

$$L << \tau_d u_h \qquad (7)$$

We assume $\tau_c^{-1} \propto T^5$ [11] so equation(6) can be integrated and we obtain the temperature of the l phonons as a function of time

$$T \,\mathrm{e}^{-\varepsilon_c/T} = T_0 \,\mathrm{e}^{-\varepsilon/T_0} \left(1 + \frac{t}{t_e}\right) \qquad (8)$$

where the effective time, t_e, is given by

$$t_e = \frac{4E_l^{(0)}(T_0) T_0 \tau_c(T_0)}{E_h^{(0)}(T_0)\varepsilon_c} \qquad (9)$$

For T_0=0.8K, $t_e \sim 10^2\tau_c$ and in this time, which is the order of 10^{-5}s, a major part of the energy has transformed into h phonons. This is short compared to typical propagation times of 10^{-4}s. So the l phonons cool as they propagate and create a trailing cloud of h phonons.

As energy is conserved between the l and h phonons, the fraction of energy in the h phonons is given by

$$\Delta(t) = \frac{E_l^{(0)}(T_0) - E_l^{(0)}(T(t))}{E_l^{(0)}(T_0)} \tag{10}$$

and equation (8) gives the temperature of the l phonons as a function of time. The fraction of energy in the h phonons for three typical starting temperatures is shown in Fig. 6. It can be seen that a significant part of the initial energy is transformed into h phonons.

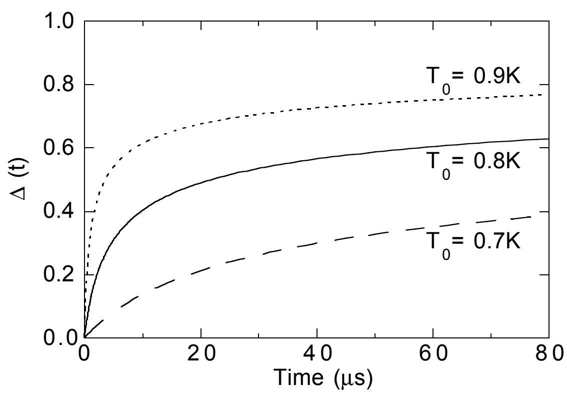

Fig. 6. The energy in the h phonons as a fraction of the initial energy as a function of time for different starting temperatures

As the l phonons cool, the creation rate of h phonons, given by equation (2), drops much faster due to the exponential factor in equation (3). So after a time $\sim 10^{-5}$s the h and l phonon pulses are relatively stable and are spatially separated. They will continue to propagate to the detector and create two distinct signals.

The flux of the h phonons is proportional to $E_l^{(0)}\mathrm{d}\Delta/\mathrm{d}t$. To calculate the pulse shape it is also necessary to include dispersion of the h phonons which we do by giving them a Bose-Einstein distribution at the temperature of the l phonons at the time they are lost from the pulse. The spectrum of h phonons then propagate with their corresponding group velocities. The computed energy flux is shown in Fig. 1 where it can be seen that it is in reasonable agreement with the measured signals.

3.2. Long pulses

For long pulses, $L >> \tau_d u_h$, the h phonon loss from the back of the pulse becomes small compared to the decay rate within the pulse so in equation (2) we can neglect the term $u_h \partial E_h/\partial z$ but in contrast to short pulses we retain the term E_h/τ_d. The h phonon density will increase with time and eventually there will be a dynamic equilibrium within the pulse. This will cause the h phonon flux to saturate which is indeed what is found: as the pulse length is increased the signal amplitude initially increases linearly but at $t_p \sim 10^{-7}$s it begins to saturate and then becomes constant.

The energy density of the h phonons in the dynamic equilibrium, $E_h^{(e)}$, is given by

$$\frac{E_h^{(e)}}{E_h^{(0)}} = \frac{\tau_d}{\tau_c} \tag{11}$$

The decay time for h phonons is very large in a pulse because of the lack of low energy phonons at the angles necessary for 4pp scattering. We estimate that τ_d can be several orders of magnitude larger than the lifetime in an isotropic equilibrium distribution. In contrast, τ_c is only a few times larger than τ_4. So we see from equation (11) that $E_h^{(e)} >> E_h^{(0)}$, that is the density becomes very much greater than an equilibrium one at the temperature of the l phonons. A supra-thermal density of h phonons is created.

For a narrow enough momentum cone angle, the decay rate due to l phonons within the l pulse tends to zero and the decay will then be due to the low energy phonons that are created coincidently with the h phonons. In this case the supra-thermal ratio $E_h^{(e)}/E_h^{(0)}$ does not depend explicitly on τ_d. To estimate the supra-thermal ratio we note that τ_4^{-1} is proportional to the number density of l phonons in the isotropic equilibrium distribution and τ_d^{-1} is proportional to the number density, m_l, of effective low energy phonons that can scatter h phonons, so

$$\frac{\tau_d}{\tau_4} = \frac{4\pi E_l^{(0)} f}{\Omega_p \bar{\varepsilon}_l m_l} \quad (12)$$

where $f < 1$ is the factor which accounts for the fact that not all the l phonon momenta in an isotropic distribution are effective in h phonon decay by 4pp.

Now if the effective low energy phonons are only created and decay simultaneously with the h phonons, i.e. we neglect other possible loss channels for the created low energy phonons, then $m_l = m_h = E_h/\varepsilon_c$. Hence with equation (12) we relate τ_d to E_h:

$$\frac{\tau_d}{\tau_4} = \frac{4\pi E_l^{(0)} \varepsilon_c f}{\Omega_p E_h \bar{\varepsilon}_l} \quad (13)$$

For the dynamic equilibrium in a long pulse we eliminate τ_d using equations (11) and (13) and find:

$$\frac{E_h^{(e)}}{E_h^{(0)}} = \left(\frac{4\pi E_l^{(0)} \varepsilon_c f \tau_4}{\Omega_p E_h^{(0)} \bar{\varepsilon}_l \tau_c} \right)^{1/2} \quad (14)$$

To estimate the magnitude of the supra-thermal ratio we take: $f = 0.5$, $\Omega_p = 0.115$sr, $E_l^{(0)}/E_h^{(0)} = 120$ for $T = 0.9$K, and $\tau_4/\tau_c = 0.3$, and find $E_h^{(e)}/E_h^{(0)} \sim 10^2$. We see that the h phonon energy density can be considerably larger than that for h phonons in a Bose-Einstein distribution at the temperature of the l phonons. This is a direct result of the lack of l phonons which can scatter and destroy the h phonons in a narrow beam of injected l phonons.

4. Conclusions

We have seen that the dynamics of phonons in a pulsed beam are very different to those in an isotropic distribution. The two main effects are: (i) the dispersion can spatially separate the high energy phonons from the low energy ones, and (ii) there is less scattering because the scattering processes are strongly angular dependent. The low and high energy phonons act as two systems that weakly interact by four phonon processes. We have solved the kinetic equation and shown how high energy phonons are efficiently created from low energy phonons. For short pulses the high and low energy phonons form into two groups which propagate ballistically. As the pulse length increases the high energy phonon flux saturates as a dynamic equilibrium is established, and because their decay lifetime is long a supra-thermal density of high energy phonons is created.

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References

[1] M Brown and A F G Wyatt, J. Phys. Condens. Matter **2** 5025 (1990).

[2] M A H Tucker and A F G Wyatt, Physica B**194-196** 549 (1994)

[3] M A H Tucker and A F G Wyatt, J. Phys. Condens. Matter **6** 2818 (1994)

[4] M A H Tucker and A F G Wyatt, J. Low Temp. Physics, **113** 621 (1998)

[5] R A Sherlock, N G Mills and A F G Wyatt, J.Phys. C Solid State Physics **8** 300 (1975)

[6] M A H Tucker and A F G Wyatt, Physica B**194-196** 551 (1994)

[7] M A H Tucker and A F G Wyatt, J. Phys. Condens. Matter **6** 2825 (1994)

[8] I N Adamenko, K E Nemchenko, A V Zhukov, M A H Tucker and A F G Wyatt, Proceedings of LT22, "The distribution function for phonons etc.".

[9] I N Adamenko, K E Nemchenko, A V Zhukov, M A H Tucker and A F G Wyatt, Proceedings of LT22, "Three phonon scattering etc."

[10] I N Adamenko, K E Nemchenko, A V Zhukov, M A H Tucker and A F G Wyatt, Phys Rev Letters **82** 1482 (1999)

[11] I N Adamenko, K E Nemchenko, A V Zhukov, M A H Tucker and A F G Wyatt, Proceedings of LT22, "Four phonon scattering etc."