

Aims and Objectives Relativity 1 and Vectors Session 3

STATIC EQUILIBRIUM; FORCES, TORQUES AND THE CENTRE OF MASS

Aims (What I intend to do)

- 1) To review rectilinear motion and the use of the vector dot product.
- 2) To examine the idea of torque and the use of the vector cross product.
- 3) To discuss the concept of centre of mass and its utility.
- 4) To look at static equilibrium and what conditions it imposes on the forces acting on an object.

Objectives (What you should be able to do after completing the lecture and worksheet)

- 1) To be able to identify situations in which the vector dot product allows the force on an object to be calculated, and to be able to do associated example problems.
- 2) To be able to identify situations in which the vector cross product allows the force on an object to be calculated, and to be able to do associated example problems.
- 3) To be able to state the conditions that apply to the forces on an object if the object is to be in static equilibrium, and to do associated example problems.
- 4) To be able to calculate the centre of mass of simple objects, and to be able to use the concept of centre of mass as a problem solving strategy.

Relativity 1 and Vectors PHY1105 Worksheet 3

- Task 1.** Go over your lecture notes and re-read sections 10.1, 10.2 and 11.1-11.3 of Young and Freedman (12th ed).
- Task 2.** Read through section 1.10 of Young and Freedman (12th ed), and do worked examples 10.1 and 10.2.
- Task 3.** Work through the derivation of the position of the centre of mass, section 8.5 of Young and Freedman.
- Task 4.** Show that by summing the torque due to each element, $\underline{\tau}_i$, where,

$$\underline{\tau}_i = \underline{r}_i \wedge \underline{w}_i$$

where \underline{r}_i is the position vector of the i^{th} element and \underline{w}_i is its weight; that the torque due to the whole object, $\underline{\Gamma}$, is given by,

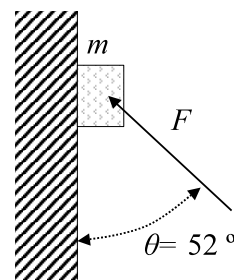
$$\underline{\Gamma} = \underline{r}_{\text{cm}} \wedge \underline{W}$$

where $\underline{r}_{\text{cm}}$ is the position vector of the centre of mass and \underline{W} is the weight of the whole object.

- Task 5.** Preparation for session 4, Read sections 6.1 - 6.3 of Young and Freedman (12th ed).
- Task 6.** Try the following question. As before, it would be a good idea to discuss this with someone else doing the module, or raise it in a tutorial.

A block having a mass of 10.0 kg is pressed against the wall by a hand exerting a force F inclined at an angle θ of 52° to the wall as shown. The coefficient of static friction μ between the block and the wall is 0.20. We will investigate the question of how large the force F must be to keep the block from sliding along the wall.

There is more physics here than initially meets the eye. Think about the situation in terms of your everyday experience (or even better, try it out). If you start with a small value for F , the block will tend to slide downward; as you increase F , you reach the point at which the block will no longer slide; as you continue increasing F , the block stays put until, at some larger value of F , it slides upward. This is the physics to be investigated, both algebraically and numerically.



- a) First draw well-separated force diagrams of the block and the region of the wall where the two are in contact (1) for the case in which F is small enough that the block tends to slide downward and (2) for the case in which the block tends to slide upward. You may wish to check section 4.6 of Young and Freedman (12th ed). Indicate the various forces by appropriate algebraic symbols; do not put in numbers at this point. (The difference between the two sets of diagrams will reside in the direction of the frictional force.) Describe each force in words and identify the third-law pairs.
- b) Applying Newton's second law, obtain algebraic expressions for F in terms of mg , μ and θ for case 1, in which the block is just about to start sliding downward and for case 2, in which it is just about to start sliding upward.
- c) Now put in the various numbers and calculate the value of F for each of the two cases. How large is the spread between the values? Does your result make physical sense? What is going on at the wall when F lies between the two extremes you have calculated? What happens to the frictional force when F lies between these two extremes?
- d) Return to the algebraic expression for case 2 in which the block is just about to slide upward. What does this expression say happens to F if you keep m and θ constant but increase the value of μ ? What is the equation telling us happens at the point at which μ is large enough to make the denominator of the expression equal to zero? Is it possible to make the block slide upward with a sufficiently large F acting at a fixed value of θ regardless of the value of μ ? Solve for the value of μ at which it becomes impossible to make the block slide upward, showing that this value depends on θ and is independent of the weight of the block. Do you find the result strange? Why or why not? Could you have anticipated it without having made the mathematical analysis?