PHY2208 Optics Problem sheet 4 Solutions

1) In the far-field, the spread due to diffraction will be well described by Fraunhofer diffraction theory, so the angular divergence of light, wavelength λ, from an aperture, diameter D, will be of order λ/D. Plugging in the numbers yields:-

$$\theta_{\text{divergence}} \approx \frac{\lambda}{D} \approx \frac{632\times10^{-9}}{0.5\times10^{-3}} \approx 1.27 \text{ milliradians}$$

After propagating 100m, the increase in diameter will be approx. $100\times1.27\times10^{-3}$ $\cong$<u>12.7cm</u>. This is much greater than the initial diameter of 0.5mm and so will be a good approximation to the actual diameter.

2) The peak power in each pulse P = pulse energy / duration, hence $P = 10^{-3} / 3\times10^{-11} = 3\times10^{7}$W. The diffraction limited spot size formed by a lens of focal ratio F is $F\lambda$ (see notes). Hence the laser beam will be focused to a spot of width 3μm i.e. area ~9×10^{-12} m^{2}. The intensity in the spot is thus $3\times10^{7} / 9\times10^{-12}$ = 3×10^{18} Wm^{-2}.

From the quoted relationship between I and E, we obtain $E=(I/(\varepsilon_0 c))^{0.5}$ = $(10^{18}/8.8\times10^{-12}/3\times10^{8})^{0.5} = 3.8\times10^{18}$ Wm^{-2}. The electric field strength in the focused spot exceeds the ionisation potential of air. This fact forms the basis of a popular demonstration in laser surgery clinics! A pulsed Nd-YAG laser beam is focused with a high-quality lens. With the lights dimmed it is easy to see blue sparks jumping out of "empty" space as the air molecules ionise and then recombine, emitting blue light in the process.

3) Sunlight has a centre wavelength of $\lambda_0 \approx 550$nm and an effective bandwidth of $\Delta\lambda$~300nm (we can see light roughly in the wavelength range 400-700nm). Hence $l_c = \lambda^2/\Delta\lambda = 550^2/300$ nm = 1210 nm i.e. <u>1.2</u> μm.
Sunlight remains coherent only over a distance of about two wavelengths ! Hence it is very difficult to observe interference effects using natural light.

4) The fringe pattern will remain stable until one or other beams undergoes a random phase jump so the fringe system will appear to shift position at a rate ~ $1/\tau_c$. These ultra-stable Helium Neon lasers (λ=543nm, $\Delta\lambda=10^{-8}$nm) then have a coherence time of <u>0.098msecs</u> (i.e. a coherence length of over. 20km!!). Hence the fringe system will appear to shift position every 0.1 millisecond or so. The student's eye will not have a fast enough response to see this (humans cannot perceive "flicker" at frequencies much above 50Hz i.e. 20msecs). The student's eye will effectively "average out" the fringes, which explains why the human eye has never directly observed interference between *separate* light sources. However, the interference between separate lasers has been observed using high-speed electronic detectors (and, of course, can be observed quite readily between separate *radio* sources).

You might like to consider the philosophical implications of observing such interference patterns, especially if the laser intensities are reduced until *only one photon at a time* travels from the lasers to the screen: - it might seem that a photon emitted by one laser must somehow be "aware" of the existence of the other laser, as its probability of reaching different parts of the screen is determined by whether the other laser is switched on or not. This is a nice illustration of the fundamentally *non-local* behaviour of quantum systems.