PHY2208 Optics Problem sheet 3 Solutions

1) Malus's law tells us that the light emerging from the first polariser is reduced in intensity by a factor two and is linearly polarised along the direction of the TA. Hence to achieve an overall attenuation of ×2, the second polariser must transmit this light with 100% efficiency. By Malus's law this requires the second TA to be parallel to the first. In b) we require an overall attenuation of ×4, so the second polariser must attenuate the light passed by the first polariser by ×2. By Malus's law this requires $\cos^2\theta = 0.5$ i.e. $\theta = 45°$.

2) At time t the rotating polariser will make an angle ωt with the TA of the first polariser. Hence by Malus' law, the rotating polariser will transmit an intensity given by:-

$$I_2 = I_1 \cos^2 \omega t$$

and since the polarisers are crossed, this emergent polarised light will make an angle ωt-90° to the TA of the second stationary polariser. Applying Malus' law a second time yields:-

$$\begin{aligned} I &= I_2 \cdot \cos(\omega t - \pi/2) \\ &= I_1 \cdot \cos^2(\omega t) \cdot \sin^2(\omega t) \\ &= I_1 \cdot \cos^2(\omega t) \cdot (1 - \cos^2(\omega t)) \end{aligned}$$

Applying the trig identity $\cos^2 \omega t = \frac{1}{2} \cdot (1 + \cos 2\omega t)$

and also in the form $\cos^2 2\omega t = \frac{1}{2} \cdot (1 + \cos 4\omega t)$

and simplifying yields the desired result:-

$$\boxed{I = \frac{I_1}{8} \cdot (1 - \cos 4\omega t)}$$

3) Recall that a perfect polariser (i.e. one with 100% transmittance for light polarised parallel to the TA and 0% transmittance for light polarised perpendicular to the TA) is expected to transmit 50% of the flux of an initially unpolarised beam. Hence imperfect polariser such as HN-32, whilst still showing 0% transmittance for light perpendicular to the TA, must only transmit 64% of the light polarised parallel to the TA.

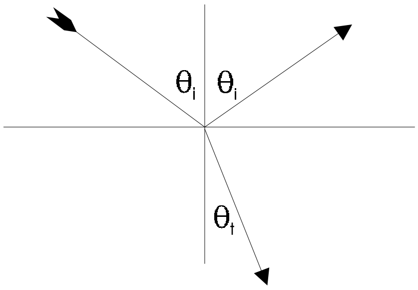
Of the incident unpolarised beam incident on the first sheet, 32% of the incident flux will be transmitted. This light will clearly be polarised parallel to the TA of the second sheet and hence will be transmitted with 64% efficiency. Hence the overall transmissivity of the two sheets is

32% × 64% = <u>20.48%</u>

4) Recall the formulae quoted in lectures for the amplitude reflection coefficients for p- and s-polarised light incident on a dielectric interface:-

$$|r_p| = \left|\frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)}\right|$$

$$|r_s| = \left|\frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)}\right|$$



Since θ_i =35° (0.6 radians) and n=1.750, Snell's law yields θ_t =19.13° (0.33 radians)

substituting into the above formula yields $|r_p|$=0.21 and $|r_s|$=0.34 for the amplitude reflection coefficients.

The intensity reflection coefficients R_p and R_s are simply the squares of the amplitude reflection coefficients i.e. R_p=0.044 and R_s=0.117

5) The incident circularly polarised beam contains p- and s- components of equal amplitude. If the reflected beam is completely s-polarised, we require $|r_p|$ = 0, which occurs when the <u>Brewster</u> condition $\theta_i + \theta_t = 90°$ is met. This can be expressed in the form
   $\tan\theta_i = \frac{n_t}{n_i}$ i.e. $\theta_i = \tan^{-1}(3.5/1.5)$ = <u>66.8°</u>

6) Recalling the formulae for the thickness of a half wave plate:-

$$d \cdot (n_e - n_o) = \left(m + \frac{1}{2}\right)\lambda$$

we see that the minimum thickness is given by:-

$$d = \frac{\lambda}{2(n_e - n_o)}$$

substituting the given values yields <u>d=0.02298mm.</u>

The quarter-wave plate has a thickness defined by

$$d \cdot (n_e - n_o) = \left(m + \frac{1}{4}\right)\lambda$$

which clearly leads to a minimum thickness which is half that of the half-wave plate i.e. <u>d=0.01149mm</u>