PHY2208 Optics Problem sheet 2
Solutions

1) The formula for the interference intensity in Young's experiment :-

$I(x) = I\cos^2\left(\frac{\pi dx}{\lambda R}\right)$ shows that the dark lines (zeros of I(x)) must be separated by a distance $\Delta x = \lambda R/d$. Substituting the values given yields the answers shown.

2) Recall the grating $m\lambda = d(\sin\theta_{out} - \sin\theta_{in)}$ equation:-

the grating dimensions imply $d=10^{-6}$m and we are told $\lambda=0.75\text{x}10^{-6}$m.
clearly since, in reflection, the term $\sin\theta_{out} - \sin\theta_{in} \leq 1$
hence $m \leq d/\lambda \leq 4/3$. m must be an integer so <u>only m=1 is possible</u>.

3) Clearly the total number of grooves N is 2000 and so, recalling R=Nm, in first order we can expect a resolution of 2000 from the grating. To separate the doublet requires:-

$$R = \lambda/\Delta\lambda = 589 / 0.6 = 982$$

Hence the grating resolution of 2000 is sufficient to resolve the lines.

4) At normal incidence, the free-spectral-range (FSR) of a Fabry-Perot, expressed in frequency, is given by:-

$$\Delta\nu_{FSR} = \frac{c}{2nd} = 3.10^8 / (2 * 0.01) = 15\text{GHz}$$

The finesse $\Im=\pi(\sqrt{F})/2$.
Recall $F=4R/(1-R)^2$ hence F=4x0.95/(0.05)^2 = 1520.
Hence $\Im=\sqrt{1520}\times\pi/2$ = <u>61.2</u>

5) recall that at normal incidence, path difference Λ for rays reflected from the upper and lower surfaces of a thin film is given by:-

$$\Lambda = 2nd$$

and that for a bright or dark fringe this should equal $m\lambda$ where m is an integer or half-integer. Then:-

$$m = 2nd/\lambda_0 = 2\times 1.5\times 2\times 10^{-3}/600\times 10^{-9} = 10{,}000$$

this is an exact integer and hence satisfies the condition for destructive interference (remembering the π phase difference between the internally and externally reflected waves). Hence the central fringe appears <u>dark</u>.

5) Michelson interferometer pathlength change = $2\Delta d$ (Δd is movement of mirror). One fringe traverses detector when pathlength changes by λ hence:-

$$15000\lambda = 2\Delta d$$
$$\Delta d = 7500\times 546.074\text{nm}$$
$$= \underline{4.096\text{mm}}$$

6) From notes:- when Michelson interferometer illuminated using two wavelengths, each produces a signal of form 1+cos(2*kd*) where *k* is propagation number $2\pi/\lambda$ and *d* is mirror displacement. Hence total signal (i.e. *incoherent* sum) is :-

$$I(d) \propto 2 + \cos\left(\frac{4\pi d}{\lambda_1}\right) + \cos\left(\frac{4\pi d}{\lambda_2}\right)$$

$$\propto 2 + 2\cos\left(2\pi d \frac{\lambda_1 + \lambda_2}{\lambda_1 \lambda_2}\right)\cos\left(2\pi d \frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2}\right)$$

Hence first minimum of fringe visibility (i.e. contrast) occurs when $2\pi d((\lambda_1-\lambda_2)/\lambda_1\lambda_2)=\pi/2$

hence $d = 0.25\ \lambda_1\lambda_2/(\lambda_1-\lambda_2)$ = <u>0.145mm</u>

7) Recall from the handout that the first minimum of the diffraction pattern from a square aperture of width *a* occurs at spatial frequency $u=\pm 1/a$.

Also, from the definition of spatial frequency $u = \frac{x}{\lambda s}$ where *x* is lateral position, λ is wavelength and *s* is the distance from aperture to observation point, we see that the first minimum of irradiance must occur at:-

$$\frac{x_{\min}}{s} \cong \frac{\lambda}{a}$$

Hence $\lambda \cong a \cdot \frac{x_{\min}}{s}$ $\cong 0.2\times\sin(36.87°)$ $\cong$ <u>12cm</u>

Recalling that the definition of “far-field” is that the aperture-observation-point distance $s >> Z_R$ the Rayleigh length a^2/λ then we require:-

$$s >> \frac{a^2}{\lambda} >> \frac{20^2}{12} \quad >> 40\,\text{cm}$$

8) From notes, the angular resolution of a telescope with a circular aperture of diameter *D* operating at wavelength λ is given by:-

$$\theta_{\min} = 1.22\frac{\lambda}{D}$$

Since D=10m and λ=600nm we have $\theta_{\min} = 1.22\times600\times10^{-9}/10 \cong 7.3\text{x}10^{-8}$ radians ($\equiv$0.015″)

Estimating the size of fine newsprint character to be ~1mm high and assuming one needs approx 10 resolution elements across a character to discern it then one needs a lateral resolution of approx 100µm.

The range at which 100µm subtends an angle of 7.3×10^{-8} radians is $10^{-4}/7.3\times10^{-8}$ = <u>1400m</u>.