PHY2208 Problem Sheet 1 Solutions

1) Concave mirror so R +ve. Real image formed 100cm away so u=+25cm and v=+100cm. Plugging these into Gaussian lens equation implies f=+20cm. Hence R (=-2f)= 40cm.

2) recalling that a mirror's focal length $f=R/2$, then since R=-80cm (the mirror is convex so has a -ve radius of curvature) f = -40cm.

Plugging this into the Gaussian lens equation along with u=+100cm yields v=-28.6cm. Recalling sign convention for mirrors, image is 28.6cm to the right of mirror vertex i.e. virtual. Magnification is $-v/u$ i.e. 28.6/100 = 0.029. $|M| < 1$ so image minified. Also M +ve so image is erect.

3) Clearly given the definitions of x_0 and x_1, the familiar object and image distances s_0 and s_1 can be expressed as $s_0 = x_0 + f$, $s_1 = x_1 + f$. Hence the Gaussian lens formula can be re-expressed as :-

$$\frac{1}{x_0 + f} + \frac{1}{x_1 + f} = \frac{1}{f}$$

i.e.

$$\frac{1}{x_0/f + 1} + \frac{1}{x_1/f + 1} = 1$$

so

$$x_0/f + 1 = \frac{f \cdot \left(1 + (x_1/f)\right)}{x_1}$$

hence

$$x_0/f = f/x_1$$

i.e.

$$x_0 \cdot x_1 = f^2$$

4) Lens maker's formula :-

$$\boxed{\frac{1}{f} = \frac{(n_2 - n_1)}{n_1} \cdot \left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

This equation implies that the focal length of a given lens will be different depending on what material it is immersed in (since n_1 will change). The ratio of the lens focal length when immersed in water, f_w, vs the focal length in air, f_a, can then be derived by calculating $1/f$ using $n_1 = 4/3$ and then $n_1 = 1$ and comparing the results. The term $(1/R_1 - 1/R_2)$ will cancel, hence if the lens glass has refractive index n_g:-

$$\frac{f_w}{f_a} = \frac{n_g - 1}{1} \Big/ \frac{n_g - n_w}{n_w}$$

substituting the values given (n_g=1.5, n_w=4/3) gives:-

$$\frac{f_w}{f_a} = \frac{(4/3)\times 1/2}{(1.5 - 4/3)} = 4$$

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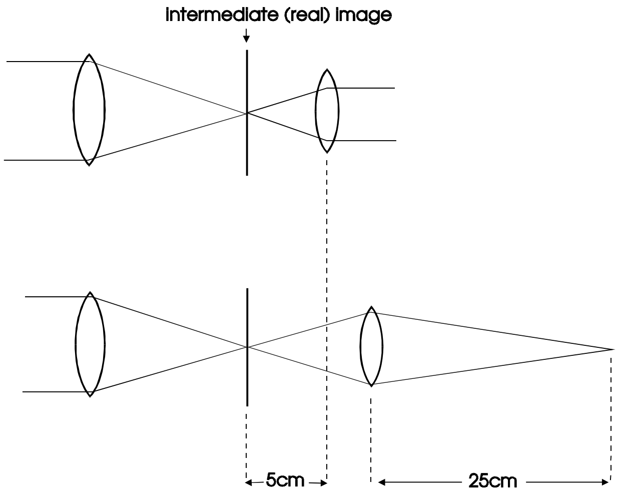
5) Remember that the astronomical telescope is usually focused so that the object appears to be at infinity. This implies that the eyepiece of 5cm focal length would usually be 5cm behind the focal plane of the objective. However, in this example the eyepiece is moved so as to focus the image of the object (as formed by the objective) onto a screen 25cm away from the eyepiece:-



For a 5cm focal length lens to produce a real image 25cm away, the lens must be a distance u from the object where:-

$$\frac{1}{u} + \frac{1}{25} = \frac{1}{5}$$

i.e. <u>u=6.25cm</u>. Hence, since the lens would normally be 5cm from the intermediate image formed by the objective, it must be moved 1.25cm away from the objective in order to form the real image as described.

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6) Since the lens has a diameter of 5cm it will have an area of approximately $1.9\text{x}10^{-3}$ m^2 . Hence it will collect a total power of 1.9W, all of which will be distributed within the projected image.

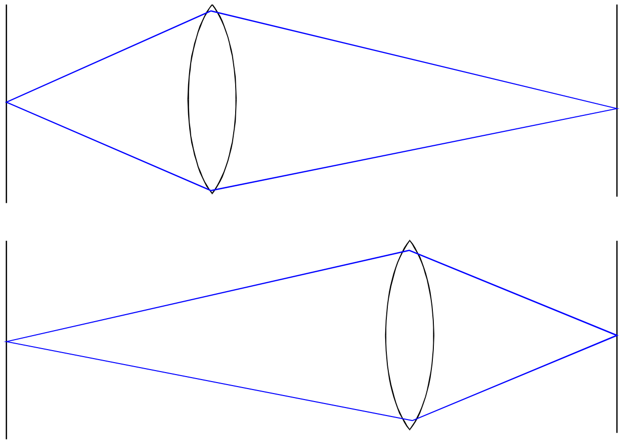
Recall that the projected image of an extended source of angular diameter θ (which is located at infinity) formed by a lens of focal length f has a physical diameter of f.θ (θ in **radians**). Hence the image of the sun formed by this lens of 0.5m focal length will have a physical diameter 0.5 x 0.5 x π/180 ≅ $4.4\text{x}10^{-3}$ metres.

so the area of the image of the sun will be $(\pi/4)\text{x}(4.4\text{x}10^{-3})^2 \cong 1.45\text{x}10^{-5}$ m^2.

The intensity of the image (total power in the image divided by the area of the image) is thus:-

intensity = (1.9)/(1.45x10^{-5}) = $\underline{1.3\text{x}10^{5}\ \text{Wm}^{-2}}$

7) For a fixed object and image location, the two positions of the lens correspond to interchanging the values of *u* and *v*:-



Since the object and image have a separation *L*, we know that the object and image distances are related by:-

$$u + v = L$$

as well as by the usual Gaussian lens equation:-

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Combining these equations yields a formula for *u*:-

$$\frac{1}{u} + \frac{1}{L-u} = \frac{1}{f}$$

rearranging this slightly:-

$$f(L-u) + fu = u(L-u)$$

i.e.

$$u^2 - uL + fL = 0$$

This is a quadratic equation for *u* and so will have two solutions (hence the lens can have two locations and still image the source onto the screen). The solutions will be given by the standard formula for the roots of a quadratic equation:-

$$u = \frac{L \pm \sqrt{L^2 - 4fL}}{2}$$

For this to have two distinct **real** solutions, we clearly require $L^2 > 4fL$ i.e. $\underline{L > 4f.}$

The separation of the two lens positions, *d*, is clearly given by the difference between these two values of *u*:-

$$d = \sqrt{L^2 - 4fL}$$

i.e. $$d^2 = L^2 - 4fL$$

so:- $$\underline{f = \frac{L^2 - d^2}{4L}}$$

8) the formula for the numerical aperture (NA) of a fibre-optic is:-

$$\text{NA } (= n_0 \cdot \sin\theta_{\text{max}}) = \sqrt{n_{\text{core}}^2 - n_{\text{cladding}}^2}$$

substituting $n_{\text{core}}=1.62$ and $n_{\text{cladding}}=1.52$ yields:-

NA=0.56.

the maximum acceptance angle ($2\theta_{\text{max}}$) in air ($n_0=1$) is thus given by:-

$$2\theta_{\text{max}} = 2 \cdot \sin^{-1}(0.56) \cong 68°$$