PHY2208 Lecture 9

Spectral resolution
The resolution of the diffraction grating

Y&F Section 38-6

Interference by amplitude division

Y&F Section 37-5
Pedrotti & Pedrotti Section 10-4

Spectral resolution describes how close two wavelengths can be and still be separated by a spectrometer.

Consider two narrow emission lines λ_1 and λ_2. If these lines can be indentified as being distinct, then the spectrometer resolution *R* is given by:

$$R = \frac{\lambda_1}{\Delta\lambda} \qquad \Delta\lambda = |\lambda_2 - \lambda_1|$$

R is a **crucial parameter** defining the performance of any spectrometer. What limits *R*? Consider $I(\phi)$ for a single λ.

Each maximum has a **finite width**, first zero is at $2m\pi+2\pi/N$ for (m=0,1,2,…). We can visualize the first zero as being when the *N* unit-length phasors wrap around to form a closed *N*-sided polygon.

If a wavelength λ_2 results in a phase $\phi = 2m\pi+2\pi/N$ while λ_1 ($> \lambda_2$) produces a phase $\phi = 2m\pi$ then we will see an intensity maximum due to λ_1 and the first intensity minimum due to λ_2 at the same diffracted angle θ_{out}. We say that these two wavelengths are just resolved. This is the **Rayleigh Criterion** (more of which later).

Hence: $$R = \frac{\lambda_2}{\Delta\lambda} = Nm$$

A typical grating has 300 lines per mm and is generally about 50mm wide:

Therefore $N = 300 \times 50 = 15000$

In 1st order $R = 15000$
In 2nd order $R = 30000$

Typical diffraction grating resolutions are thus about 10^4 in first order.

Prism spectrometers are limited to resolutions of about 600 (limited by **diffraction** - see later).

Interference by amplitude division

Multiple-slit interference is **interference by wavefront division**.

Interference by amplitude division involves splitting a wavefront by reflection/refraction at a refractive index mismatch.

e.g. coloured fringes due to oil on water.

Consider a plane wavefront incident on a plane sheet (ref. index *n*) immersed in a medium of refractive index 1.0.

At each interface, electromagnetic wave theory demands that both a reflected and a transmitted wave is generated. The incident, reflected and transmitted waves must be **coplanar**.

Optical path difference $= 2nd\cos\theta_t$

When OPD is $m\lambda$, **destructive** interference is produced.

Why destructive? The wave reflected at the first interface suffers a **π phase change** relative to that reflected at the second.

If plane waves strike the interface over a range of θ_i and a lens focuses them, we will see concentric rings of different colours:

These are **fringes of equal inclination** - a fringe is a locus of constant θ_i

Now attempt Questions 2-4 on Problem Sheet 2