

## PHY2208 Lecture 8

- Interference patterns in Young's experiment
- Complex phasors
- $N$ -slit interference
- The diffraction grating

Y&F Sections 37-1 to 37-4, p. 1174, 38-6  
Pedrotti & Pedrotti Section 10-2

Now let's resolve the problem of 2-slit interference by using a graphical construction known as a **vibration curve**.

This method immediately generalizes from two-slit interference to  $N$ -slit interference.

$N$ -slit interference is exploited by the **diffraction grating** (a dispersive element which can replace the prism in a spectrometer).

The complex amplitude at P,  $\Psi_P$ , is the sum of two complex amplitudes of equal modulus, but possessing phase difference  $\phi$ :

$$\begin{aligned}\Psi_P &= A[\exp(i(kr_1 - \omega t)) + \exp(i(kr_1 - \omega t + \phi))] \\ &= A \exp(i(kr_1 - \omega t))(1 + \exp(i\phi))\end{aligned}$$

Ultimately we are interested in  $|\Psi_P|^2$  so the term  $A \exp(i(kr_1 - \omega t))$  contributes a constant factor  $A^2$ . We then only need to find the modulus of  $1 + \exp(i\phi)$ .

Physically, interference depends only on the **relative phase** between the waves, not on their absolute phases.

We can determine this modulus graphically using an Argand diagram:

OR and RS are **phasors** of unit modulus. Consider the isosceles triangle OQR with  $\angle OQR = \phi$ . OQR and RQS are **congruent**.

So

$$\begin{aligned}I(x) &= |\Psi_P|^2 = A^2 |OS|^2 = 4I \cos^2((kd \sin \theta) / 2) \\ &\approx 4I \cos^2\left(\frac{\pi dx}{\lambda R}\right)\end{aligned}$$

as before.

Consider a screen containing a large number ( $N$ ) of slits. Assuming the same geometry as before, what interference pattern will we observe?

The vibration curve method generalizes immediately to give

$$|\Psi(\phi)| = \frac{A \sin(N\phi/2)}{\sin(\phi/2)} \quad I(\phi) = \frac{A^2 \sin^2(N\phi/2)}{\sin^2(\phi/2)}$$

In the limit  $\phi \rightarrow 0$ ,  $\sin(N\phi/2)/\sin(\phi/2) \rightarrow N$ . Hence the peak intensity due to  $N$  slits is a factor of  $N^2$  greater than due to 1 slit.

Clearly  $I(\phi)$  is *periodic*, with a period of  $2\pi$ .  $I(\phi)$  is zero when  $\phi = 2m\pi/N$ .

Therefore for large  $N$ ,  $I(\phi)$  displays **narrow, pronounced** maxima with a period  $2\pi$ .

Since  $\phi = kd \sin \theta$ , this predicts a strong transmitted beam in a direction given by:

$$\sin \theta = m \frac{\lambda}{d}$$

where  $m$  is an integer. Hence this array of slits has dispersive properties similar to the prism.

Such an element is constructed in several ways:

- Laying fine wires onto a glass sheet
- Scratching lines into a metal-coated glass sheet
- Use 'photolithography' - (also used to make integrated circuits)

In practice, diffraction gratings are often used in reflection geometry and are illuminated by a **collimated** beam.

Each thin metallic strip scatters the incoming plane wavefront and gives rise to a spherical wavefront.

Strip 2 receives the incoming plane wavefront with a **phase lag** relative to strip 1 of

At the field point P, the wavefront from strip 2 arrives first, i.e. with a **phase lead**:

The overall phase difference between the complex amplitudes due to strip 1 and strip 2, at P is then

The condition for an intensity maximum in this direction is still  $\phi = 2m\pi$  hence:

$$\sin \theta_{\text{out}} - \sin \theta_{\text{in}} = \frac{m\lambda}{d}$$

This is called the **grating equation**.

This equation tells us that for a given angle of illumination  $\theta_{\text{in}}$  and a given grating **pitch** (i.e.  $d$ ), at what angle  $\theta_{\text{out}}$  will a given wavelength  $\lambda$  produce an intensity maximum.

If  $m=0$  then  $\theta_{\text{out}} = \theta_{\text{in}} \Rightarrow$  law of reflection.

If  $d < \lambda$  this is the only solution, as  $\sin \theta_{\text{out}} - \sin \theta_{\text{in}} < 1$ . Hence a mirror is a diffraction grating whose pitch is less than  $\lambda$ !

The grating reflects light at angles  $\theta_{\text{out}} \neq \theta_{\text{in}}$  and  $\theta_{\text{out}}$  depends on  $\lambda$ .

The diffraction grating thus produces a dispersed spectrum when illuminated by polychromatic light.

$m$  is **any integer**, hence more than one spectrum is produced (provided  $\theta_{\text{out}} < 90^\circ$ ). For  $m$  equal e.g. 0, 1, 2 we speak of the grating's **zeroth order spectrum, first order spectrum etc**

The zeroth order spectrum **is not dispersed** and hence deprives useful spectra ( $m \neq 0$ ) of light, which reduces the efficiency of the spectrometer.

Spectra can **partially overlap** (e.g.  $\lambda=800$  nm produces an  $m=1$  maximum at the same angle as  $\lambda=400$  nm produces an  $m=2$  maximum) and the spectrum can get confused.

Now attempt Questions 1 & 2 on Problem Sheet 2.