PHY2208 Lecture 7

The prism spectrometer (concluded)
Interference

Y&F Section 34-5
Pedrotti & Pedrotti p. 116-120
Pedrotti & Pedrotti p. 200-204

Since glass is dispersive, n is λ-dependent, and so θ is λ-dependent.

The **prism spectrometer** uses a prism and two lenses to record the spectrum of a **polychromatic** point source:

The linear dispersion of a spectrograph is:

$$\frac{d\lambda}{dx} = \left(f \frac{d\theta}{d\lambda} \right)^{-1} = \left(\alpha f \frac{dn}{d\lambda} \right)^{-1}$$

where f is the focal length of the focusing lens.

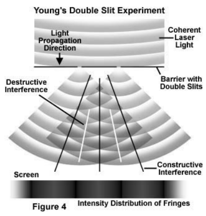
$\frac{dn}{d\lambda}$ is known as the **dispersion** of the glass (typically in nm^{-1})

e.g. crown glass @ 500nm has $\frac{dn}{d\lambda} = 8 \times 10^{-5}\,\mathrm{nm}^{-1}$

Interference

This effect proves definitively that light is a wave.

Young (1802) performed a simple experiment whose results cannot be explained by ray optics:



Wave model predicts that the wave incident on the slits spreads into two new spherical waves centred on the holes. These combine and **interfere** on the screen.

Simplify by considering the interference pattern formed by two separate sources. We describe the wave amplitude due to each of the sources in isolation by a complex amplitude:

$$\Psi_1(r,t) = \frac{A \exp i(kr - \omega t)}{r}$$

NB the real part of this function physically represents the **electric field amplitude** due to the travelling EM wave emanating from the source.

EM waves are governed by the principle of **linear superposition**.

The complex amplitude Ψ_p due to two source of equal amplitude At distances r_1 and r_2 from point of interest P:

$$\Psi_P = \Psi_1 + \Psi_2 = \frac{A \exp i(kr_1 - \omega t)}{r_1} + \frac{A \exp i(kr_2 - \omega t)}{r_2}$$

We see the intensity of light, and this is given by:

$$I_P = |\Psi_P|^2 = \Psi_P \Psi_P^*$$

(Physically we detect energy density of the EM wave; energy density is proportional to $|\mathbf{E}|^2$.)

Then:

$$I_P = A_1^2 + A_2^2 + 2A_1A_2 \cos[k(r_2 - r_1)]$$

where $A_1 \equiv A / r_1$ etc

We express the intensity at a point P due to the interference of two sources as:

$$I_P = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$$

Where $\phi = k(r_2 - r_1)$ is the **phase difference** between the interfering Waves. $(r_2 - r_1)$ is the **path difference**.

To calculate an explicit form for the interference pattern observed in Young's experiment we make two approximations:

1) The pattern is observed on a screen whose distance from the slits R is **large** compared to their separation d. The straight lines joining the slits to P are then effectively **parallel**.

2) These lines make an angle θ (see diagram) which is small.

Although r_1 and r_2 clearly differ, if $R >> d$ the effect on the denominator of the complex amplitudes Ψ_1 and Ψ_2 can effectively be ignored. Hence we can write $A_1 \approx A_2$ $(= A)$.

For a point P at distance x from O, the intensity is given by:

For small θ $\sin\theta \approx x / R$ hence

$$I(x) = 4I\cos^2\left(\frac{\pi d x}{\lambda R}\right)$$

Since k = 2π/λ

So we see 'cos-squared' fringes formed on the screen. The intensity varies between 0 and 4 times the intensity due to one slit alone (I). Such fringes are typical of two-source interference.