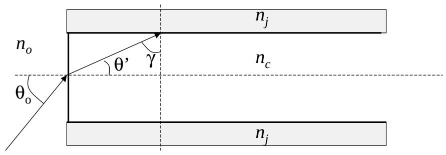
Lecture 6

- Fibre optics (concluded)
- Fermat's principle of least time

- The wave theory of light
- Spectroscopy
- The prism spectrometer

Y&F Sections 34-2 and 34-5
Pedrotti & Pedrotti p. 119-122

Fibre optics (continued)



Here n_o is the ref. index of the medium external to the fibre (air), n_c is that of the fibre core, and n_j is that of the **jacket** or **cladding**. Using the same procedure as before we get:

$$n_0 \sin\theta_0 = \sqrt{n_c^2 - n_j^2}$$

Fermat's principle of least time

According to Fermat's principle, *light takes the path of least time.*

This is an important theorem from which **all** laws of reflection and refraction can be derived.

The wave theory of light

Light is a transverse electromagnetic (EM) wave. A point source of light emits spherical EM waves. The **wavefronts** expand at speed c. 'Rays' merely denote the *local normal* to a wavefront at any point in space:

The propagation of light is thus described by a wave equation:

$$\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = 0$$

Where $\psi(\mathbf{r}, t)$ represents the strength of an electric field at $(\mathbf{r}, t)$. This equation has many solutions. One especially useful one is:

$$\psi(r,t) = A\frac{\exp i(kr - \omega t)}{r}$$

Describing a spherical wave propagating away from the origin with speed ω/k. r is the radial distance from the origin, ω is the angular frequency and k is the propagation number $=2\pi/\lambda$ where λ is the wavelength.

Another useful solution is:

$$\psi(\mathbf{r},t) = A\exp i(\mathbf{k}.\mathbf{r} - \omega t)$$

Describing a plane wave propagating in the direction **k**, with a speed $\omega/|\mathbf{k}|$. **r** denotes a position in space. In both cases A is an arbitrary scalar amplitude.

Visible light encompasses roughly the wavelength range 400nm to 700nm, with the shortest wavelengths appearing blue and the longest red. 500nm appears green.

'White light' is composed of a continuum of wavelengths thoughout the visible spectrum.

Spectroscopy

Spectroscopy is the technique whereby 'white' (more exactly **polychromatic**) light is analysed to determine the amount of each wavelength present.

If $I(\lambda)d\,\lambda$ is the light intensity of a polychromatic source within the infinitesimal wavelength range λ to $\lambda+d\,\lambda$, then $I(\lambda)$ is the **spectrum** of the source.

Spectrometers are devices to measure $I(\lambda)$.

Isaac Newton obtained a crude spectrum of sunlight using a simple prism spectrometer:

Consider light rays refracted symmetrically through a prism. A simple wave-optics argument can be used to derive the **deviation** θ.

Wave theory requires that wavefronts be continuous everywhere. Therefore the wavefronts must match up as a wave transverses a refractive boundary.

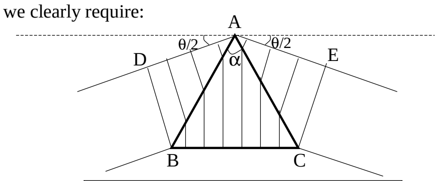
Note that within the prism (ref. index n) the wavelength is λ/n.

The optical path length (OPL) in a medium with refractive index n is defined as

OPL = physical path $\times\, n$

Hence the same number of wavefronts always fit into a given OPL, regardless of the medium.

To ensure that the wavefronts match up after traversing the prism, we clearly require:



OPL (along BC) = OPL (along DA+AE)

Hence $n \times$ BC = DA +AE

Simple geometry then implies:

i.e.

For a small prism angle and small deviations, this reduces to:

$$n \approx \frac{\theta + \alpha}{\alpha} \quad \text{i.e.} \quad \theta = \alpha(n-1)$$