PHY2208 Lecture 4

- Refraction at a spherical interface
- Thin lenses
- The lens maker's formula

- Pedrotti & Pedrotti Section 3-8 & 3-9
- Y&F 35-6

Images can also be formed at spherical interfaces between media with different refractive indices.
Consider a convex spherical interface between media with $n=n_a$ and $n=n_b$:

Since PVP' strikes at normal incidence it us undeviated. It thus Suffices to determine where ray PBP' crosses the optic axis.

As with the derivation for a spherical mirror we:
1) Express s and s' in terms of $\tan\alpha$ and $\tan\beta$

2) Note that the paraxial approximation applies:

3) Use simple geometry to relate the angles:

4) We now apply Snell's Law at B to yield:

5) Make the paraxial approximation to reduce Snell's Law to

Expressions in step 3 become:

6) Eliminating θ_a yields:

7) Substituting from step 2 and cancelling *h* yields:

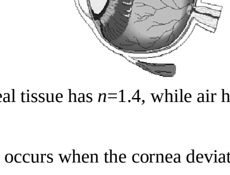
$$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{(n_b-n_a)}{R}$$

Object-image relation for a spherical refracting surface.

Note how *h* cancels, so **any** ray from P which strikes the interface will pass through P' i.e. P' is a **real image** of P. Also note the similar form to the equivalent relationship for the spherical mirror.

Example

As well as protecting the eye, the cornea helps focus light (in fact it provides about 70% of the refractive power of the eye).



The corneal tissue has n=1.4, while air has n=1.0.

Astigmatism occurs when the cornea deviates from a spherical shape.

The lens maker's formula

Lenses are made by placing two such interfaces in proximity. The **thin-lens** is an abstraction – we consider that the spacing between the interfaces is negligible.

Consider a thin spherical lens as two spherical interfaces, radii of curvature R_1 and R_2, formed between media with refractive indices n_a, n_b, and n_c.

Consider an object P at distance s_1 from the first interface. According to the object-image relation the first interface forms an image P' at distance s'_1 where:

$$\frac{n_a}{s_1} + \frac{n_b}{s'_1} = \frac{(n_b - n_a)}{R_1}$$

We can consider the image P' to represent an object at distance $s_2 = - s'_1$ from the second interface.

The second interface forms an image of P' (labelled P'') at a Distance s'_2. Using the object-image relation:

$$\frac{n_b}{s_2} + \frac{n_c}{s'_2} = \frac{(n_c - n_b)}{R_2}$$

Substituting $s_2 = - s'_1$ (valid for thin lens) these equations yield

$$\frac{n_a}{s_1} + \frac{n_c}{s'_2} = \frac{(n_b - n_a)}{R_1} + \frac{(n_c - n_b)}{R_2}$$

For a lens $n_c = n_a$. Also writing s_1 as s and s_2' as s' we obtain:

$$\frac{n_a}{s} + \frac{n_a}{s'} = (n_b - n_a)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

i.e.

$$\frac{1}{s} + \frac{1}{s'} = \frac{(n_b - n_a)}{n_a}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

This resembles the Gaussian mirror equation. i.e. we have

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Gaussian lens equation

$$\frac{1}{f} = \frac{(n_b - n_a)}{n_a}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

The lens maker's formula

f positive $\Rightarrow$ converging lens

f negative $\Rightarrow$ diverging lens

NB this lens possess two focal points (front and back) equidistant from the lens.

To trace rays through lenses we choose any two of:

1) A ray which passes through the **optical centre** O is undeviated.

2) A ray which enters the lens having passed through the front focal point exits parallel to the optical axis.

3) A ray which enters the lens parallel to the optic axis exits so as to pass through the back focal point.

You are now in a position to tackle problems 1 to 4 of Problem Sheet 1.