

PHY2208 Lecture 14

Describe Fraunhofer diffraction in terms of the **2-D Fourier Transform**

Pedrotti & Pedrotti Chapter 16 Into, 16-1, 16-2

Fraunhofer diffraction is an **extremely** important case:

Since

$$\Psi(x, y) \propto \iint_{\text{aperture}} T(\xi, \eta) e^{-2\pi i f(\xi, \eta)/\lambda} dA(\xi, \eta)$$

and when $s_0 \gg Z_R$

$$f(\xi, \eta) \approx \frac{x\xi + y\eta}{s_0}$$

hence

$$\Psi_p \propto \iint_{\text{aperture}} T(\xi, \eta) \exp(-2\pi i(x\xi + y\eta)/(\lambda s_0)) d\xi d\eta$$

We now introduce new variables to describe the observation point P

$$u = x/(s_0\lambda)$$

$$v = y/(s_0\lambda)$$

Hence

$$\Psi_p \propto \iint_{\text{aperture}} T(\xi, \eta) \exp(-2\pi i(u\xi + v\eta)) d\xi d\eta$$

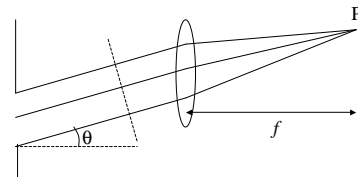
Since $T(\xi, \eta) = 0$ outside the aperture the integral can be extended to the range $\pm\infty$.

$$\Psi_p \propto \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} T(\xi, \eta) \exp(-2\pi i(u\xi + v\eta)) d\xi d\eta$$

The complex amplitude of a Fraunhofer diffraction pattern is the 2-D Fourier Transform of the aperture transmission Function.

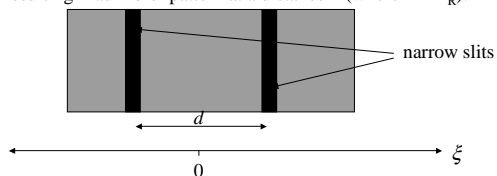
NB To observe Fraunhofer diffraction, points in the aperture must contribute to Ψ_p with phases which vary **linearly** with their position in the aperture. This is called the **Fraunhofer condition**.

The Fraunhofer condition can also be achieved in another way:



The complex amplitude at the focus of an aberration-free +ve lens is the 2-D Fourier Transform of the lens transmission function.

Consider two long slits of negligible width and separation d . Using Fourier theory we can work out the intensity pattern of the resulting Fraunhofer pattern at a distance L (where $L > Z_R$):



Since the slits are long we can consider the system as having one dimension so

$$\Psi_p(x) = \int_{-\infty}^{\infty} T(\xi) \exp(-2\pi i u \xi) d\xi$$

where $u = x/(L\lambda)$

The aperture transmission function T is made up of two **Dirac delta functions**, δ .

The delta function has the following properties:

$$\delta(x) \begin{cases} = \infty, & x = 0 \\ = 0, & x \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(x) dx = 1$$

And for a continuous function $f(x)$

$$\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$$

$$\int_{-\infty}^{\infty} \delta(x-a) f(x) dx = f(a)$$

So we have:

$$T(\xi) = \delta\left(\xi + \frac{d}{2}\right) + \delta\left(\xi - \frac{d}{2}\right)$$

$$\Psi_p(x) = \int_{-\infty}^{\infty} \delta\left(\xi + \frac{d}{2}\right) \exp(-2\pi i u \xi) d\xi + \int_{-\infty}^{\infty} \delta\left(\xi - \frac{d}{2}\right) \exp(-2\pi i u \xi) d\xi$$

It can be shown (see derivation):

$$I(x) = |\Psi_p(x)|^2 \propto \cos^2\left(\frac{\pi x d}{\lambda L}\right)$$