

PHY2208 Lecture 13

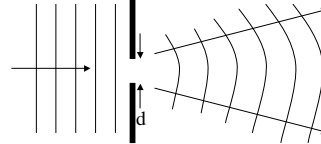
Diffraction (the uncertainty principle)
 Diffraction (the Huygens-Fresnel principle)
 Define **Fraunhofer** and **Fresnel** diffraction
 Define the **Rayleigh distance**

Y&F Section 37-7, 38-1 and 38-2
 Pedrotti & Pedrotti Chapter 16 (introduction)

Diffraction is a term used to describe how a wavefront is altered when made to pass either

- a) through an aperture in a screen, or
- b) around an obstruction.

Consider case (a) i.e. a plane wave impinges on an opaque screen, which contains an aperture of diameter D :



After propagating through the aperture, the wavefront becomes curved i.e. a lateral spread in the direction of energy transfer is produced.

A simple quantum mechanical argument can predict the principle result of the more rigorous treatment:

In 1905 Einstein explained the photoelectric effect by considering that the energy in a light wave is quantized into units of hc / λ . These quanta of energy are **photons**.

In 1905 Einstein developed special relativity, and predicted that $E = pc$ for a photon, hence $p = h / \lambda$.

Around 1921 Heisenberg postulated the Uncertainty Principle:

$$\Delta x \Delta p \approx h$$

i.e. laterally confining a particle to a region of width Δx induces an associated uncertainty in momentum (in direction x) of $\approx h / \Delta x$.

A simplistic quantum view of diffraction is thus of photons passing through the aperture and being confined to a region of width D .

If the plane wave has wavelength λ , and we call z the direction of propagation:

$$p_z = \frac{h}{\lambda}$$

The aperture of width D induces an uncertainty in momentum in the transverse direction (x) of

$$\Delta p_x = \frac{h}{D}$$

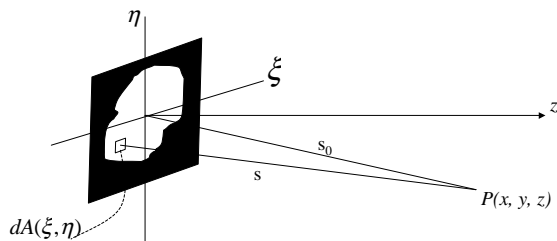
Hence an uncertainty in the angle of propagation $\Delta\theta$ is induced where

$$\Delta\theta \left(= \frac{\Delta p_x}{p_z} \right) \approx \frac{\lambda}{D}$$

i.e. an initially plane wave is diffracted by ever greater amounts as λ increases or D decreases. This places a **fundamental limit** on the optical performance of lenses and mirrors.

We can develop a more sophisticated treatment, based on classical wave theory:

Consider the following geometry:



The **Huygens-Fresnel principle** states that the complex amplitude P can be calculated by considering each point within the aperture to be a source of spherical waves.

The **aperture transmission function** $T(\xi, \eta)$
 $0.0 \equiv$ opaque; $1.0 \equiv$ 100% transmitting

Consider an elemental area $dA(\xi, \eta)$ in the aperture a distance s from $P(x, y, z)$

$$\Psi_p \propto \iint_{\text{aperture}} T(\xi, \eta) \frac{e^{i(ks - \omega t)}}{s} dA$$

In realistic geometries we can make various **simplifying assumptions**:

- a) The variation due to the term $1/s \ll$ that due to $\exp(i(ks - \omega t))$, so we can replace $1/s$ term by a constant factor $1/s_0$.
- b) Derive a power series expression for $s(x, y, \xi, \eta)$:

$$s \approx s_0 - \frac{x\xi + y\eta}{s_0} + \frac{\xi^2 + \eta^2}{2s_0}$$

If we denote terms other than s_0 by $f(\xi, \eta)$ – **the aperture phase function**:

$$\Psi(x, y) \propto e^{-i\alpha x} e^{2\pi i s_0 / \lambda} \iint_{\text{aperture}} T(\xi, \eta) e^{-2\pi i f(\xi, \eta) / \lambda} dA(\xi, \eta)$$

$$|\Psi(x, y)| \propto \iint_{\text{aperture}} T(\xi, \eta) e^{-2\pi i f(\xi, \eta) / \lambda} dA(\xi, \eta)$$

Usually $s_0 \gg x, y$ and $s_0 \gg \xi, \eta$ so it is usual to ignore all but the first two terms in $f(\xi, \eta)$ i.e.:

$$f(\xi, \eta) \approx \frac{x\xi + y\eta}{s_0} - \frac{(\xi^2 + \eta^2)}{2s_0}$$

$f(\xi, \eta)$ describes the phase with which $dA(\xi, \eta)$ contributes to $\Psi_p(x, y)$

NB the first term in the above expression describes a contribution to the total phase which varies **linearly** with the aperture coordinates ξ, η . The second term describes a contribution that varies **quadratically**.

Given that $s_0 \gg x, y$ and $s_0 \gg \xi, \eta$ we saw that the phase with which $dA(\xi, \eta)$ contributes to $\Psi_p(x, y)$ is given by $-2\pi f(\xi, \eta) / \lambda$ where

$$f(\xi, \eta) \approx \frac{x\xi + y\eta}{s_0} - \frac{(\xi^2 + \eta^2)}{2s_0}$$

We define two classes of diffraction pattern:

a) Fresnel diffraction, where both terms contribute significantly to $f(\xi, \eta)$

b) Fraunhofer diffraction, where the quadratic term is negligible

Consider a 1-D aperture of maximum length a (i.e. a slit). The criterion in (b) can be formally expressed as:

$$\frac{2\pi}{\lambda} \frac{a^2}{2s_0} \ll \pi$$

Hence to observe Fraunhofer diffraction we require

$$s_0 \gg \frac{a^2}{\lambda}$$

The characteristic length a^2/λ is the **Rayleigh distance** Z_R

For an aperture of **arbitrary** shape:

$s_0 \gg Z_R$ defines the **far-field**
 $s_0 \sim Z_R$ defines the **near-field**