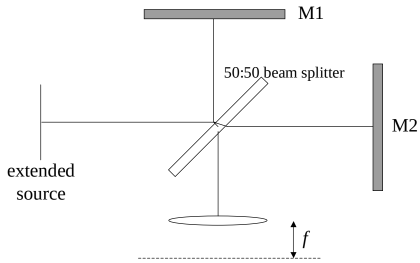
PHY2208 Lecture 12

The Michelson interferometer

Pedrotti & Pedrotti Section 11-1 & 11-2

One can produce 'virtual' thin-film interference using two 100% reflective mirrors:

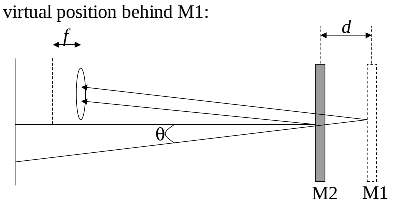


The beamsplitter is a 'half-silvered' mirror: glass with a thin (< λ) metal coating. This yields R = 50% and T = 50%.

Consider any rays leaving the extended source. The corresponding wavefront is amplitude-divided by the beam-splitter. These two wavefronts traverse different arms and reflect off M1 and M2 respective (M1 and M2 have R = 100%)

50% of each of these wavefronts pass through the beam splitter to reach the lens. They will interfere in the lens focal plane.

We can simplify the arrangement by considering M2 to lie in a virtual position behind M1:



Hence we obtain a 'virtual' thin film which produces fringes of equal inclination (see Lecture 9).

Consider the point P, subtending angle θ to the mirror-normal. The OPD between the two rays is

$$\text{OPD} = 2d\cos\theta$$

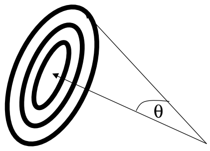
where d is the difference in length between the two arms of the interferometer.

We obtain an intensity maximum at angles θ which satisfy:

$$2d\cos\theta = m\lambda + \alpha$$

(α arises from phase shifts at the beam splitter. Let us assume α=0)

Hence (assuming M1 and M2 are perfectly parallel) we see concentric circular fringes:



Since two equal amplitude beams produce (see Lecture 7)

$$I_P = 2I_0 + 2I_0\cos[k(r_2 - r_1)]$$

at the centre of the pattern we have:

$$I(\theta = 0) = 2I_0 + 2I_0\cos(2kd)$$
$$= 4I_0\cos^2\left(\frac{2\pi d}{\lambda}\right)$$

By scanning one mirror, one obtains a cos-squared variation of intensity. Zeros of intensity occur at

$$d = \left(m + \frac{1}{2}\right)\frac{\lambda}{2}$$

By counting how many fringes one traverses during a mirror scan of length D, on can infer λ (e.g. Michelson measured λ for a cadmium emission line to be 643.84722 nm by scanning one mirror through 1 m and 'counting' 3, 106, 327 fringes!)

Now consider applying light from two narrow emission lines λ_1 and λ_2 to the device.($\lambda_2 > \lambda_1$).

At $d = 0$, both λ's (and many other λ) produce constructive interference and we see 'white light' fringes.

As d increases, eventually we will have:

$$d = \left(m + \frac{1}{2}\right)\frac{\lambda_1}{2}$$

$$d = m\frac{\lambda_2}{2}$$

i.e. the maximum due to λ_2 coincides with a minimum due to λ_1

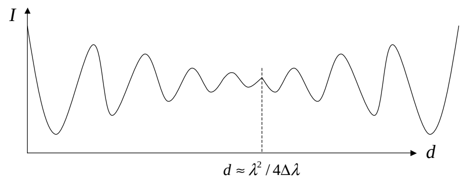
We see a minimum in the contrast of the fringe pattern:

$$d = \frac{\lambda_1 \lambda_2}{4(\lambda_2 - \lambda_1)} \approx \frac{\lambda^2}{4\Delta\lambda}$$

Defining the fringe contrast via the **visibility function**:

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

V has its first zero at this value of d :



Fourier transform spectroscopy (P&P p. 536-539) generalizes this technique and uses a Michelson interferometer to measure an arbitrary spectrum. $\Re$ is determined by the total distance scanned by M1.