PHY2208 Lecture 10

The Fabry-Perot interferometer

Pedrotti & Pedrotti p. 233-244

The **Fabry-Perot interferometer** consists of two highly reflective, plane parallel surfaces, possibly enclosing a medium of different refractive index to that outside the surfaces.

Also known as an **etalon** (especially if the plate separation is fixed).

High reflectivity means many reflected/transmitted beams will interfere. As with the diffraction grating, this produces high spectral resolution.

Consider a plane wave, incident at angle θ_i hence refracted at angle θ_t . Transmitted ray (2) experiences an OPD (see thin film section in Lecture 9):

$$OPD = 2nd\cos\theta_t$$

and hence an additional phase

$$\phi = \frac{4\pi nd}{\lambda}\cos\theta_t$$

Also, ray (2) suffers 2 additional reflections.

Denote the **amplitude** reflection coefficient r and the transmission coefficient t. For simplicity we will assume these are real quantities.

If incident complex amplitude is Ψ_0, then the total transmitted amplitude Ψ_t is

$$\Psi_t = (\Psi_0 t)t + (\Psi_0 t)r^2 e^{i\phi} t + (\Psi_0 t) r^4 e^{2i\phi} t + \ldots$$

This is a geometric series with a first term $\Psi_0 t^2$ and a common ratio $r^2 e^{i\phi}$.

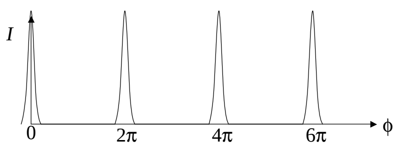
By summing this series to infinity, and writing R (the **intensity** reflection coefficient) = r^2 , T (**intensity** transmission coefficient) $=t^2$, we obtain:

$$I(\phi) = I_0 \frac{T^2}{(1-R)^2}\frac{1}{1+F\sin^2(\phi/2)}$$

Where F (the coefficient of finesse) is determined by the mirror reflectivity R via

$$F = \frac{4R}{(1-R)^2}$$

NB the denominator is periodic with period $2m\pi$ (c.f. diffraction grating) and $I(\phi)$ has maxima at $\phi = 2m\pi$, m=0,1,2,...



As F (i.e. R) increases, these maxima get narrower.

The Fabry-Perot thus has wavelength dispersive properties similar to the diffraction grating, and can be used for spectroscopy

Spectral resolution is again determined by the width of the maxima. However $I(\phi)$ never falls to zero, hence we cannot apply the Rayleigh criterion to calculate the resolution $\Re$.

The **Taylor Criterion**: two spectral lines can just be separated if the separation equals the full-width at half-maximum (FWHM) of each individual line

---

It is more convenient to work in frequency:

$$\Re = \frac{\lambda}{\Delta\lambda} = \frac{\nu}{\Delta\nu}$$

$$\Re = \frac{\nu}{\Delta\nu} = \frac{\pi}{2}\sqrt{F}m$$

This is also written $\Re = \Im m$, where $\Im (= \pi F^{1/2} / 2)$ is the **finesse**. Compare this with $\Re = Nm$ for the diffraction grating.

Now attempt Question 5 on Problem Sheet 2 (but not the free-spectral range bit yet).