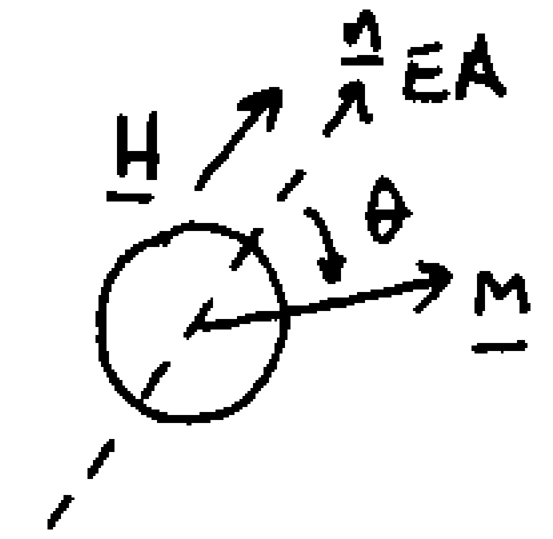
1.



SW model: the particles are uniformly magnetized and the magnetic free energy density per unit volume may be written as

$$E = -\underline{M}\cdot\underline{B} - K(\hat{\underline{M}}\cdot\underline{n})^2 = -\mu_0\underline{M}\cdot\underline{H} - K(\hat{\underline{M}}\cdot\underline{n})^2$$
$$= -\mu_0 MH\cos\theta - K\cos^2\theta$$

The orientation of the magnetization corresponds to an energy minimum which we find by setting

$$\frac{\partial E}{\partial\theta} = \mu_0 MH\sin\theta + 2K\cos\theta\sin\theta = 0$$

which has solutions $\theta = 0, \pi, \cos^{-1}\left(-\frac{\mu_0 MH}{2K}\right)$

For these extrema to be minima, we require

$$\frac{\partial^2 E}{\partial\theta^2} = \mu_0 MH\cos\theta + 2K\cos 2\theta > 0$$

If H is large and positive we expect $\theta = 0$ to be the minimum that is occupied. As the field is reduced and then reversed the minimum remains stable until

$$\frac{\partial^2 E}{\partial\theta^2} = 0, \text{ at which point } \mu_0 MH + 2K = 0$$
$$H = -\frac{2K}{\mu_0 M} \text{ is the switching field.}$$

For switching $B = \mu_0 H = -\frac{2K}{M} = -\frac{2\times 6\times10^5}{480\times10^3} = \underline{2.5\,T}$

The lifetime of the written state is given by an Arrhenius law $\tau = \tau_0\exp(E_b/kT)$, $E_b = KV = \frac{4\pi}{3}r^3 K$
$\tau_0 \simeq 1\,ns$

At $T = 300K$, $k_B T = 26$ meV, $E_b = \frac{4\pi}{3}\times 27\times10^{-27}\times 6\times10^5$
$= 6.8\times10^{-20}\,J = 0.425\,eV$

$$\therefore \tau = 10^{-9}\exp\left(\frac{0.425}{0.026}\right) = 10^{-9}\exp(16) = \underline{13\,ms}$$

But for data to be stable for 10 years $\tau = 10\times365\times24\times3600$
$= 3.2\times10^8\,s$

$$\therefore E_b/kT = \ln\left(\frac{3.2\times10^8}{10^{-9}}\right) = 40$$
$$E_b = 1.0\,eV, \quad r = \left(\frac{3E_b}{4\pi K}\right)^{1/3} = \underline{4.1\,nm}$$

2.

Co
AlOx
Co

$$\text{TMR} = \frac{2P_1P_2}{1-P_1P_2} = \frac{2P^2}{(1-P^2)}$$

where

$$P = \frac{g_\uparrow(E_F) - g_\downarrow(E_F)}{g_\uparrow(E_F) + g_\downarrow(E_F)} = \frac{(g_\uparrow/g_\downarrow) - 1}{(g_\uparrow/g_\downarrow) + 1} = \frac{r-1}{r+1}$$

So, if $\text{TMR} = 0.65$, rearranging

$$(\text{TMR})(1-P^2) = 2P^2$$

$$(\text{TMR}) = (2+\text{TMR})P^2$$

$$P^2 = \frac{\text{TMR}}{2+\text{TMR}} = \frac{0.65}{2.65}$$

$$P = \sqrt{\frac{0.65}{2.65}} = \underline{0.50}$$

Rearranging again,

$$(r+1)P = r-1$$

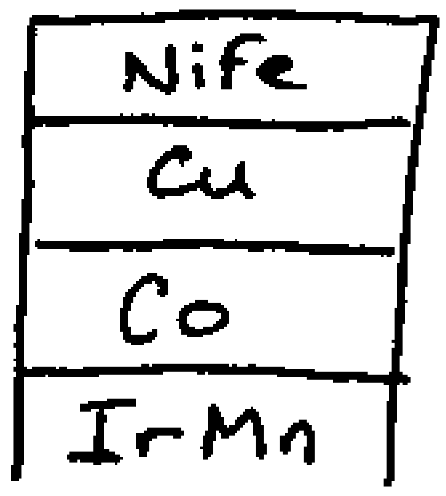
$$r(P-1) = -(P+1)$$

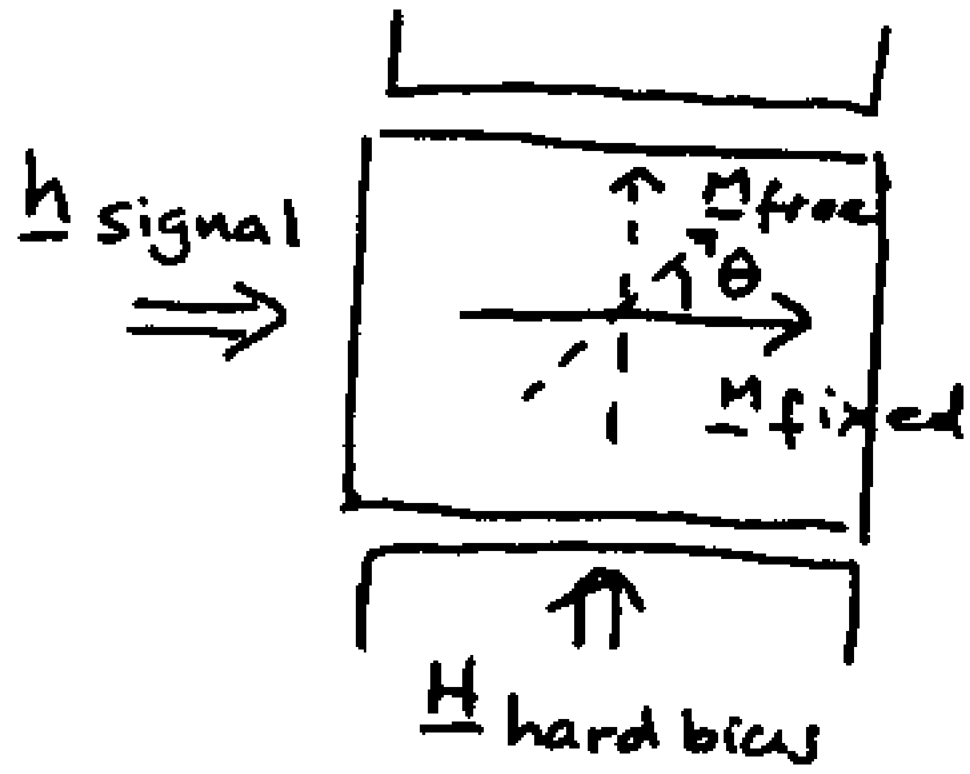
$$r = \frac{g_\uparrow}{g_\downarrow} = \left(\frac{P+1}{1-P}\right) = \frac{1.5}{0.5} = \underline{3}$$

TMR might be further increased by using:

- higher spin polarization electrodes eg. Heusler alloys that approach half-metallic behaviour
- expitaxial tunnel barriers where the symmetry of the electrode wave fn must match that of the evanescent barrier state.

3.

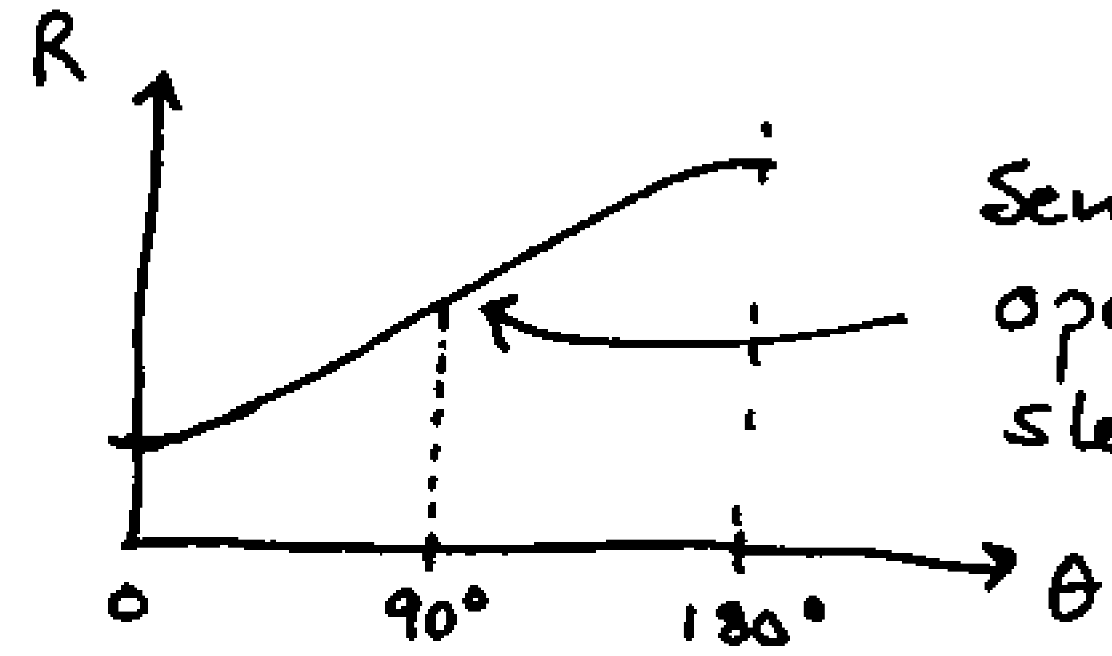




$GMR = 0.08$

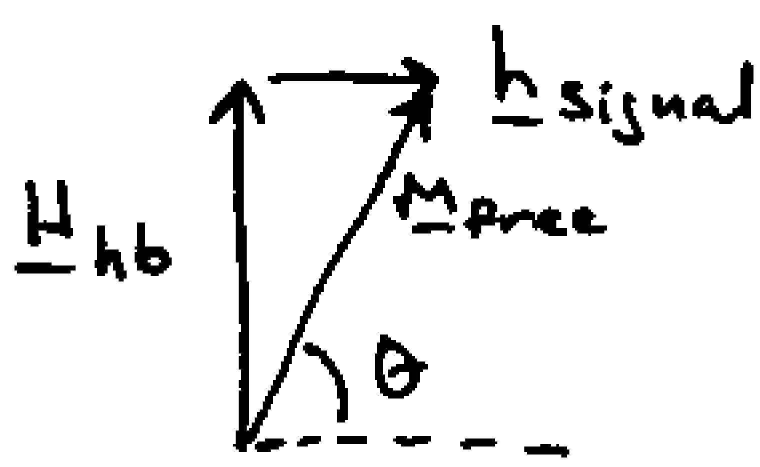
$|\underline{H}_{hb}| = 1\ kA/m$

$$GMR = \frac{R_{AP} - R_P}{R_P} \qquad R = R_{AP} + R_P \mp (R_{AP} - R_P)\cos\theta$$



Sensor biased to operate here where slope greatest.

When the signal field is applied, $\underline{M}_{free}$ rotates to align with $\underline{H}_{hb} + \underline{h}_{signal}$



for small signal field,

$$\cos\theta = \frac{h_{signal}}{H_{hb}}$$

$\therefore$ the resulting change in resistance is

$$\Delta R = -(R_{AP} - R_P)\,\Delta(\cos\theta) = -(R_{AP} - R_P)\frac{h_{signal}}{H_{hb}}$$

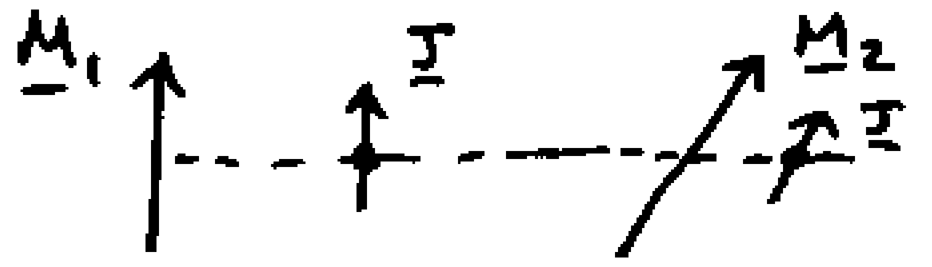
$$\therefore \left(\frac{\Delta R}{R_P}\right) \Big/ h_{signal} = -\frac{(R_{AP} - R_P)}{R_P} \Big/ H_b$$

$$= \underline{0.08\%\ \text{per}\ kA/m}$$

4.

Assume that electrons injected from $\underline{M}_1$ into $\underline{M}_2$ carry angular momentum parallel (anti-parallel to be accurate) parallel to $\underline{M}_1$.

But, angular momentum of electrons must become aligned with $\underline{M}_2$ after the electrons have travelled some distance in $\underline{M}_2$.



So the angular momentum of the injected electrons has been modified and this requires a torque to be exerted on $\underline{M}_2$.

$$\Gamma = \Delta \underline{J} = \underline{J} - (\underline{J}\cdot\underline{\hat{M}}_2)\,\underline{\hat{M}}_2$$

$$\propto \underline{\hat{M}}_1 - (\underline{\hat{M}}_1\cdot\underline{\hat{M}}_2)\,\underline{\hat{M}}_2 = (\underline{\hat{M}}_2\cdot\underline{\hat{M}}_2)\underline{\hat{M}}_1 - (\underline{\hat{M}}_1\cdot\underline{\hat{M}}_2)\underline{\hat{M}}_2$$

$$= \underline{\hat{M}}_2 \times (\underline{\hat{M}}_2 \times \underline{\hat{M}}_1)$$

from the BAC-CAB rule: $\underline{A}\times(\underline{B}\times\underline{C}) = (\underline{B}\cdot\underline{A})\underline{C} - (\underline{C}\cdot\underline{A})\underline{B}$

Hence $\underline{\Gamma \propto \underline{M}_2 \times (\underline{M}_2 \times \underline{M}_1)}$

Spin transfer torque driven motion occurs when the STT exceeds the damping in this configuration. Ignoring contributions other than the applied field to $\underline{B}_{eff}$, $\underline{B}_{eff} = B\,\underline{\hat{M}}_1$ and so we require

$$-\frac{\alpha}{M_2}\,\underline{M}_2 \times (\underline{M}_2 \times B\,\underline{\hat{M}}_1) - \frac{g\mu_B J}{2|\gamma||e|M_2^2 d_2}\,\underline{M}_2 \times (\underline{M}_2 \times \underline{\hat{M}}_1) = 0$$

i.e.
$$-\frac{\alpha B}{M_2} - \frac{g\mu_B J}{2|\gamma||e|M_2^2 d_2} = 0$$

$$|\gamma| = \frac{|e|}{2m} = \frac{1.6\times10^{-19}}{2\times9.1\times10^{-31}} = 8.8\times10^{10}$$

$$J = \frac{-2\alpha B|e|M_2 d_2|\gamma|}{g\mu_B}$$

$$|J| = \frac{2\times0.01\times4\pi\times10^{-7}\times10^{+3}\times1.6\times10^{-19}\times5\times10^{5}\times2\times10^{-9}\times8.8\times10^{10}}{2\times9.27\times10^{-24}}$$

$$= \frac{4\pi\times1.6\times8.8}{9.27}\times10^{6} = \underline{1.9\times10^{7}\ \text{A/m}^2}$$