

PHY2009 Physics of Crystals:

Problems Sheet for lectures 7 and 8

- 1) A 2D direct lattice has primitive lattice vectors:

$$\mathbf{a} = a\mathbf{i}$$

$$\mathbf{b} = b((\cos\alpha)\mathbf{i} + (\sin\alpha)\mathbf{j})$$

[note: these are just vectors of lengths a and b with angle α between them]

- (a) Sketch a portion of this lattice, indicating the cartesian axes and the primitive lattice vectors.
- (b) Show that the reciprocal lattice has primitive vectors:

$$\mathbf{a}^* = \frac{2\pi}{a} \left(\mathbf{i} - \frac{\cos\alpha}{\sin\alpha} \mathbf{j} \right)$$

$$\mathbf{b}^* = \frac{2\pi}{b} \frac{1}{\sin\alpha} \mathbf{j}$$

[Note: the 2D reciprocal lattice can be determined in two ways:

EITHER: write \mathbf{a}^* and \mathbf{b}^* as general vectors in 2D and then determine the values of their x and y components that allow them to satisfy the Laue condition;

OR: use the formulae relating 3D reciprocal and direct lattice vectors, but use $\mathbf{c} = c\mathbf{k}$ for the third direct lattice vector and allow c to tend to infinity. The third reciprocal lattice vector will then be of length zero, and can simply be ignored.]

- (c) Sketch the reciprocal lattice indicating its orientation with respect to the direct lattice.
- (d) Sketch both the direct and reciprocal lattices in the case $a = 0.1$ nm, $b = 0.2$ nm, $\alpha = 60^\circ$. [Hint: they should be rectangular.]
- (e) On your sketch of the rectangular reciprocal lattice above, construct the first and second Brillouin zones. [Note: you know how to construct the first one. The second Brillouin zone is the region of k -space (not necessarily a single region) reached from the chosen reciprocal lattice point, by crossing one, and only one, perpendicular bisector – it should have the same 'volume' as the first Brillouin zone.