PHY2009 Physics of Crystals:

Problems Sheet for lectures 16 to 18

1) Potassium is a monovalent free-electron metal with an atomic weight of 39.

(a) Calculate the number density of electrons, given that the density of potassium is 860 kg m^{-3}.

(b) By integrating the density of states function (or otherwise) find the Fermi energy of potassium.

(c) What proportion of the electrons have energies within $k_B T$ of E_F at room temperature.

(d) Estimate the electronic contribution to the room-temperature heat capacity of potassium and compare this with the classical value, $3k_B/2$ per electron.

(e) Calculate the Fermi velocity, and compare this

(i) with the electron's thermal velocity at room temperature

(ii) with the drift velocity of an electron in a potassium wire of length 10 m, cross-sectional area 1mm^2, when a voltage of 10 V is applied across it. (the resistivity of potassium is 6.15×10^{-8} Ωm)

2) Show that the total kinetic energy of a Fermi gas of N free electrons, at 0 K, is

$$U_0 = \frac{3}{5} N E_F$$

3) Calculate the Fermi energy, and Fermi temperature, of

(a) liquid ^{3}He (density 81 kg m^{-3})

(b) neutrons in a neutron star (density 10^{17} kg m^{-3})

4) According to the nearly-free electron model, aluminium behaves like a free-electron metal, *except* that its electrons have an effective mass (which is not equal to the bare electron mass).

Aluminium has atomic weight 27, and density 2700 kg m^{-3}. calculate the number density of free electrons.

The Fermi energy of aluminium is 12 eV. Calculate the effective mass.

The electrical resistivity of aluminium at 300 K is 3×10^{-8} Ωm. Calculate the mean free path of the conduction electrons, and their mean drift velocity in an electric field of 1000 V m^{-1}.

5) Show that the number of (particle-in-a-box) k-states in the first Brillouin zone of a simple cubic free-electron metal is equal to the number of lattice sites.

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