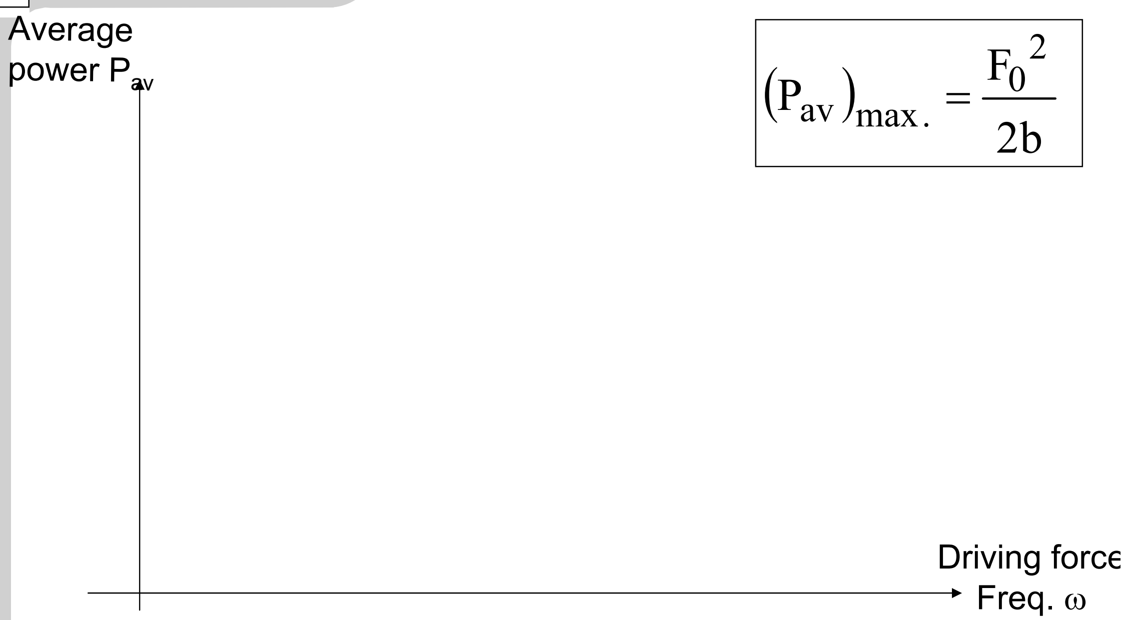
Power resonance curve (leading to Q-factor)

$$\text{Average power}: \; P_{av} = \frac{F_0^{\,2}}{2|Z_m|}\cos\phi$$

This average power is a **maximum** when;

i.e. in conditions of resonance; when

- ………. and …………… are in phase
- the ……………… equals the ………… freq. of system
- ………….. has a minimum value = ………



$$(P_{av})_{max.} = \frac{F_0^{\,2}}{2b}$$

(We call ω_1 and ω_2 the frequencies at which the P_{av} is ………………………….. value.)

• The peak power transferred to a system occurs at the …………………. **resonance**

i.e. (when ……. and ……… are exactly in phase),

• this is represented in the **absorption curve** (shown previously).

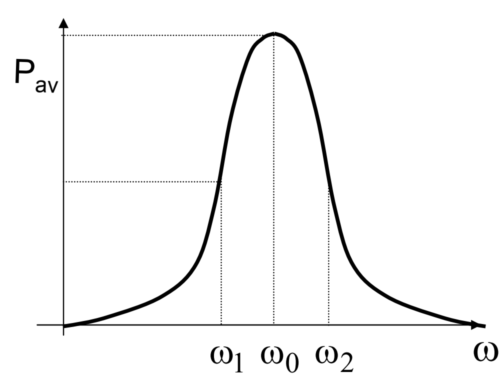
Note that the **power resonance** (and the velocity resonance) occurs at frequency;

Q-value for a resonant system.

The Q-value tells us …….
……………………………..

High Q means very ……..
…………………….

Definition:



Where for, ω_1, ω_2; $P_{av} = \frac{1}{2}\left(P_{av}\right)_{max}$

We can now derive an expression for Q that is more useful than the one above. (Pain p65)

Start with these two equations, that we've met already;

$$P_{av} = \frac{bF_0^{\,2}}{2Z_m^{\,2}} \quad (1) \qquad (P_{av})_{max.} = \frac{F_0^{\,2}}{2b}$$

Remember this expression for the mechanical impedance:

$$Z_m^{\,2} = b^2 + \left(\omega m - k/\omega\right)^2 \quad (2)$$

At frequencies that correspond to ω_1, and ω_2,

$$P_{av} = \tfrac{1}{2}(P_{av})_{max.} = \tfrac{1}{2}.\frac{F_0^{\,2}}{2b} \quad (3)$$

So we can equate equations 1 and 3 then simplify. $\longrightarrow$ $Z_m^{\,2} = 2b^2$

Now substitute equation 2;

$$2b^2 = b^2 + \left(\omega m - k/\omega\right)^2$$

And simplify;

$$b^2 = \left(\omega m - k/\omega\right)^2$$

$$\longrightarrow \quad \omega m - \frac{k}{\omega} = \pm b$$

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Assuming that $\omega_2 > \omega_1$ for this equation $\quad \omega m - \frac{k}{\omega} = \pm b$

then;

$$\omega_2 m - \frac{k}{\omega_2} = +b \qquad \omega_1 m - \frac{k}{\omega_1} = -b$$

$$\omega_2^{\,2} m - k = +b\omega_2 \qquad \omega_1^{\,2} m - k = -b\omega_1$$

$$\left(\omega_2^{\,2} - \omega_1^{\,2}\right)m = b(\omega_2 + \omega_1)$$

$$(\omega_2 + \omega_1)(\omega_2 - \omega_1)m = b(\omega_2 + \omega_1)$$

$$(\omega_2 - \omega_1) = \frac{b}{m}$$

$$Q = \frac{\omega_0}{\omega_2 - \omega_1}$$

$$Q = \frac{\omega_0 m}{b}$$

As the retarding force b decreases, Q (the sharpness of the resonance) increases

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End of section II

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Dr. Pete Vukusic, Exeter University:
PHY 1106: Waves and Oscillators (Lecture 8)

Section III

Alternating Electrical Currents (Steady state)

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Dr. Pete Vukusic, Exeter University:
PHY 1106: Waves and Oscillators (Lecture 8)

III.1. Initial ideas

Voltage:

AC voltages ……………………………………………

But due to the nature of the components, there may be a …………………….. between the ……… and the …………….

Current:

To calculate the current flowing in circuits, we use a version of ……………………..

By using exponential notation, we can see the effect of ………………………..

Resistor:

This works in the same way as for DC, and remains a component that is **real-only**.

Capacitor:

The freq. of ……………. affects the size of ……………… (and its phase) flowing through the capacitor.(**Complex**).

Inductor:

The frequency of ………… also affects the ……………. (and its phase) through an inductor. (**Complex**).

Representing driving voltage.

This is our alternating voltage.