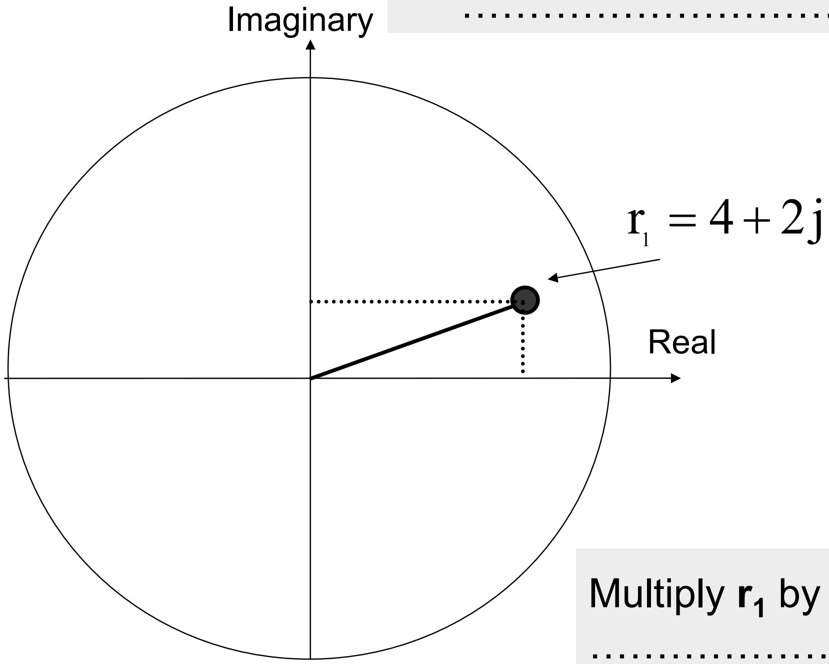
i) Multiply $\mathbf{r_1}$ by **j** and we produce $\mathbf{r_2}$.

ii) The difference between r_1 and r_2 is ……………………………………..



Multiply $\mathbf{r_1}$ by **-j** and you …………………….. by …… degrees (…….. radians)

II.2 Mechanical impedance Z_m

In advance of the analysis, we have to define a quantity known as …………………………….

It is a complex quantity (real and imaginary parts), and represents the ……………………… to the ………. that tries to get it moving.

Formally;

In words; **the mechanical impedance is the** ……… **required to produce** ……………………..

To simplify the way we present the complex nature of the impedance,

Complex number

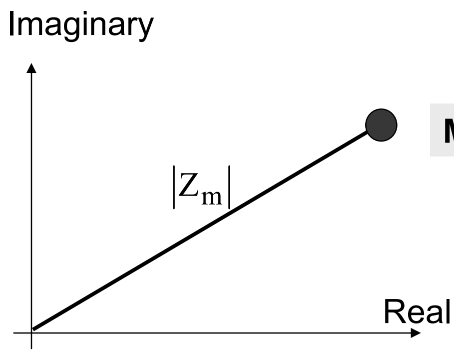
Modulus

Phase term

From one of our definitions of mechanical impedance, we can calculate its phase.

Real part

Imag. part



Modulus

Phase angle of impedance

Forced oscillations analysis

Set up equation and solve for complex amplitude of motion

$$\hat{x} = \hat{A}e^{j\omega t}$$

Substitute back to find expression for displacement

Appreciate the phase between displacement and driving force

Displacement (amplitude) resonance

Substitute back to find expression for velocity

Appreciate the phase between velocity and driving force

Velocity resonance

II.3 Analysis of forced oscillation

We represent the driving force in exp(jωt) form.

The equation becomes;

As a solution, we try the following equation;

(Where x and A are now complex numbers).

by substitution;

From this; we find an expression for the complex amplitude;

Now, multiply top and bottom by -j;

Substitute in for mechanical impedance;

Now substitute this amplitude back into expression for displacement;

ie. into $\hat{x} = \hat{A}e^{j\omega t}$

Now substitute in the polar representation for mechanical impedance;

$$\hat{Z}_m = |Z_m| e^{j\phi}$$

And we have;

Simplifying;

This gives us all the information we need to predict what happens to the ………………………..

The complex equation gives us the following information;

$$\text{Displacement} = \hat{x} = -j\frac{F_0 e^{j(\omega t - \phi)}}{\omega |Z_m|}$$

i) It completely defines the …………….. and ………. of the displacement (x) w.r.t the driving force F.

ii) A ……………………. exists between x and the driving force F.

iii) There is an additional phase difference introduced by the ……………… term.

(even if ϕ=0; the …………… would …………… the …………………….. by 90 degrees).

Remember: we used $F_0 \cos\omega t$ and represented it by ……….

So we take the ……….. part of x;

So taking the real part of the displacement, x

$$\text{Real}(x) =$$

Note:

The maximum amplitude of the displacement is

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Summary so far, for "*displacement*".

$$\text{Re}(x) = \frac{F_0}{\omega|Z_m|}\sin(\omega t - \phi)$$

The displacement lags the force by *90 degrees* *plus the phase factor* ϕ**.**

So even if ϕ=0,

$$x = \frac{F_0}{\omega|Z_m|}\sin(\omega t)$$

but remember

……. degree ……… difference

But what happens at low and high frequency?

(Still II.3) Behaviour of displacement versus frequency

expression for displacement; $x = \frac{F_0}{\omega |Z_m|} \sin(\omega t - \phi)$

The bit at the beginning is the amplitude of the displacement; ie.

Displacement amplitude =

We need to look at how the mechanical impedance varies with frequency, but first we must take its modulus;

If; $Z_m = b + j\left(\omega m - k/\omega\right)$

then;