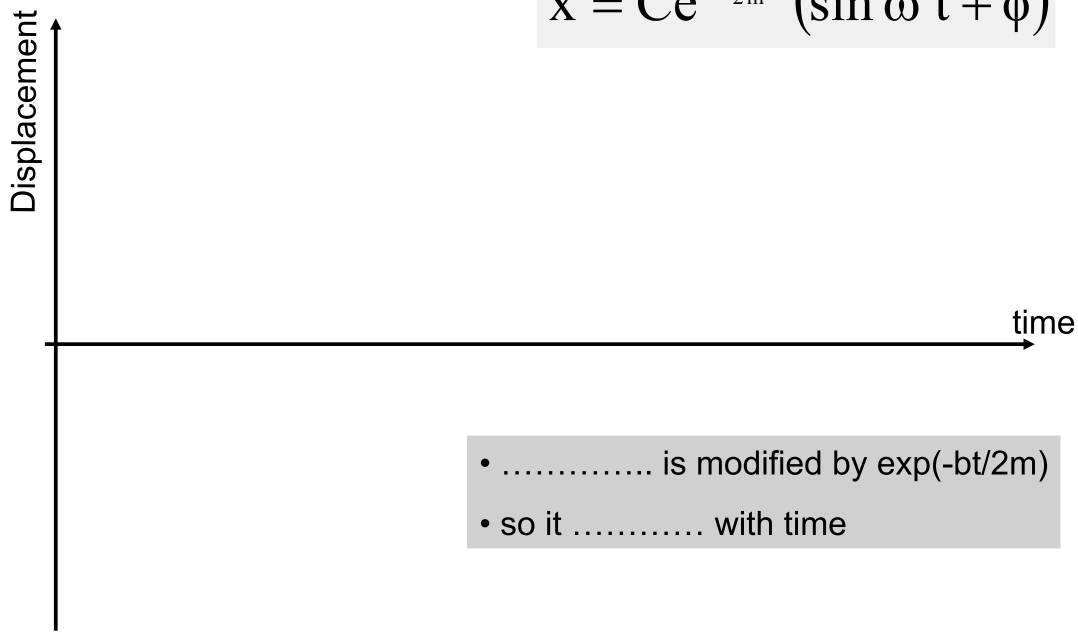
So …..

The displacement varies sinusoidally with time but with a new frequency ω'

$$x = Ce^{-\frac{b}{2m}t}\left(\sin\omega' t + \phi\right)$$



- …………. is modified by exp(-bt/2m)
- so it ……….. with time

(Diagram drawn with $\phi = 0$)

Points to note;

1. **‘C’ is the amplitude for motion with no damping.**
2. **‘b’ is the damping coefficient of the system**
3. **Since** $\text{energy} \propto \text{amplitude}^2$ **then**

$$\text{energy} \propto \left(e^{-\frac{b}{2m}t}\right)^2$$
$$\propto e^{-\frac{b}{m}t}$$

This tells us how the energy decays!

The *logarithmic decrement* provides a variable which tells us how quickly the motion decays.

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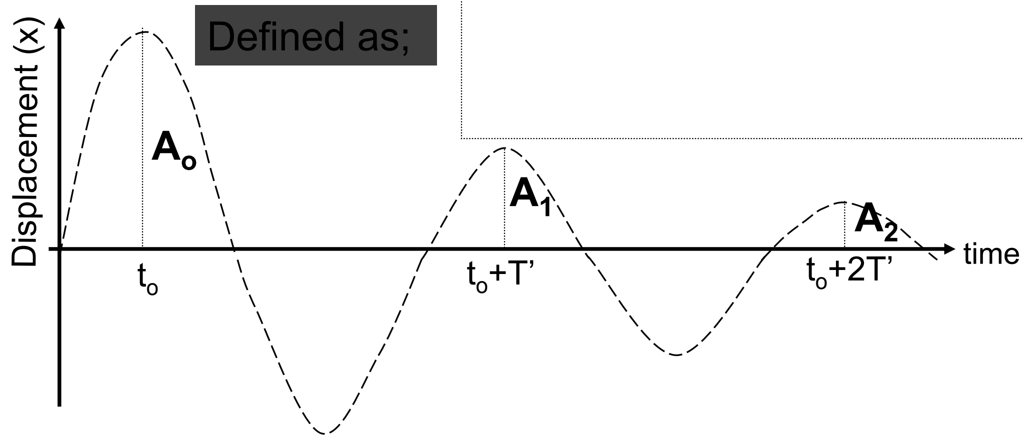
(Pain p35)

Logarithmic decrement: δ

Defined as;



From previous equations, we can write the following;

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Now, taking the ratio associated with the logarithmic decrement;

$$\frac{x_0}{x_1} =$$

This simplifies to; $\frac{x_0}{x_1} =$

And so finally; $\delta =$

ie. δ is the logarithmic decrement with;

δ is the logarithm of the ratio of the amplitudes over a period.

Points to note;

1. The formula also reduces to $\frac{x_0}{x_n} = e^{\frac{nb}{2m}T'}$

So a plot of **log$_e$ (x_0/x_n)** versus **n** gives a straight line of slope bT'/2m.

$$\log_e \frac{x_0}{x_n} = n \frac{b}{2m} T' \longrightarrow y = mx$$

I.5 Quality factor or Q-value of a damped SH oscillator

The Q-factor gives a measure of the ………………………….. in the system.

(Note. If little energy is lost per cycle, then Q will be high).

From other theory;

$$E \propto (\text{amplitude})^2 \propto$$

Therefore;

The ……….. in the system decays with time.

We can produce a simpler equation for the Q-factor.

Start with; $E = E_0 e^{-\frac{b}{m}t}$

Differentiate;

Substitute and re-arrange;

But we call dt = T'; which is time for one full period

Remember; $Q = 2\pi . \frac{\text{energy in system}}{\text{energy lost per cycle}} =$

Remember also; $\omega' = \frac{2\pi}{T'}$

So finally;

We could also show that;

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Energy dissipation in damped SHM

For no damping, the energy in a system is constant.

$$E = KE + PE = \text{constant}$$

$$E = \tfrac{1}{2}m\dot{x}^2 + \tfrac{1}{2}kx^2$$

But with damping, the energy decreases with time.

$$\frac{dE}{dt} \neq 0$$

Differentiating the energy expression;

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2\right)$$

$$= \dot{x}m\ddot{x} + xk\dot{x}$$

$$= \dot{x}\left(m\ddot{x} + xk\right)$$

(But previously we had $m\ddot{x} + kx = -b\dot{x}$ **)**

So; $\frac{dE}{dt} = -\dot{x}(b\dot{x}) = -b\dot{x}^2 = -bv^2$

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$$\frac{dE}{dt} = -bv^2$$

The right side of the equation is <u>always negative</u>, even when the velocity is positive.

This means that the energy continually decreases.

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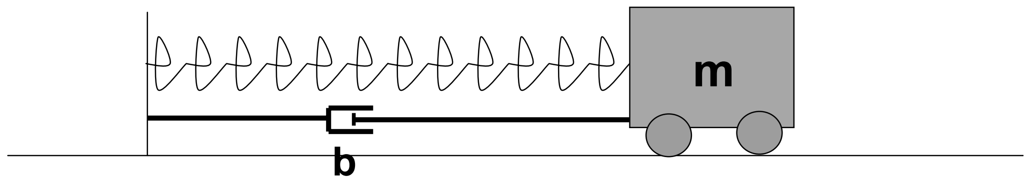
End of Section I

II. 1 The Forced Oscillator

Consider the situation;



The spring / mass system;

- i) Spring constant ……….
- ii) velocity damping term ……...
- iii) and driving force …………..

Balancing forces for this situation;

$$m\ddot{x} + b\dot{x} + kx = F_0 \cos\omega t$$ → Equation of motion for a forced oscillator

The complete solution consists of two contributions.

1. A ………… term;	This part of the solution has been done already. (The solution to $m\ddot{x} + b\dot{x} + kx = 0$) It comprises ……….. and …………...
2. ……………. term;	This describes the …………….. behaviour of the oscillator.

In this section we are interested in the <u>steady state</u> term!

Ie. The solution to $m\ddot{x} + b\dot{x} + kx = F_0 \cos\omega t$

Complex numbers and phase.

Harmonic oscillators can be conveniently represented by exp(jωt).

e.g. for a.c. driving voltages,

$$V_0 e^{j\omega t}$$

e.g. for sinusoidal forces,

$$F_0 e^{j\omega t}$$

In addition to mathematical convenience, j is an operator which is associated with ………….

i.e. multiply a complex number (or any number) by j and you …………. its phase by ….. degrees (…… radians).