Total Energy term for SHM

Total energy = KE + PE

For undamped motion, this total energy E is …….

N.B. at max. displacement, PE = maximum and KE = 0;

- but this applies at all times,
- since amplitude …….

A more rigorous derivation of; $\left(\frac{1}{2}kA^2 = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2\right)$

$x = A\sin(\omega t + \phi)$ $\dot{x} = A\omega\cos(\omega t + \phi)$

$E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$

But $\omega^2 = \frac{k}{m}$

Total energy at any time, displacement, velocity or acceleration is .....

Parabolic form of potential energy

Start with this function; $\frac{1}{2}kA^2 = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$

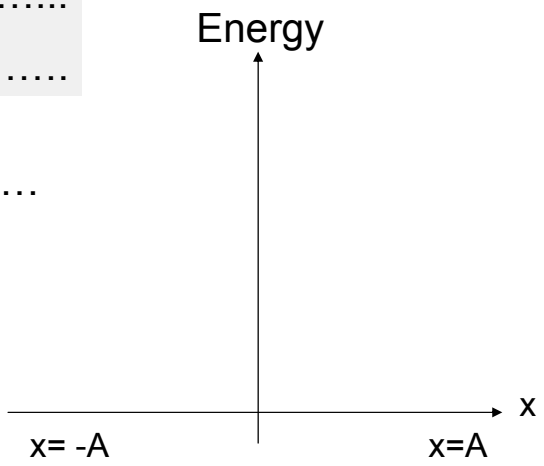
Re-arrange to find ……

(Recognize this!)

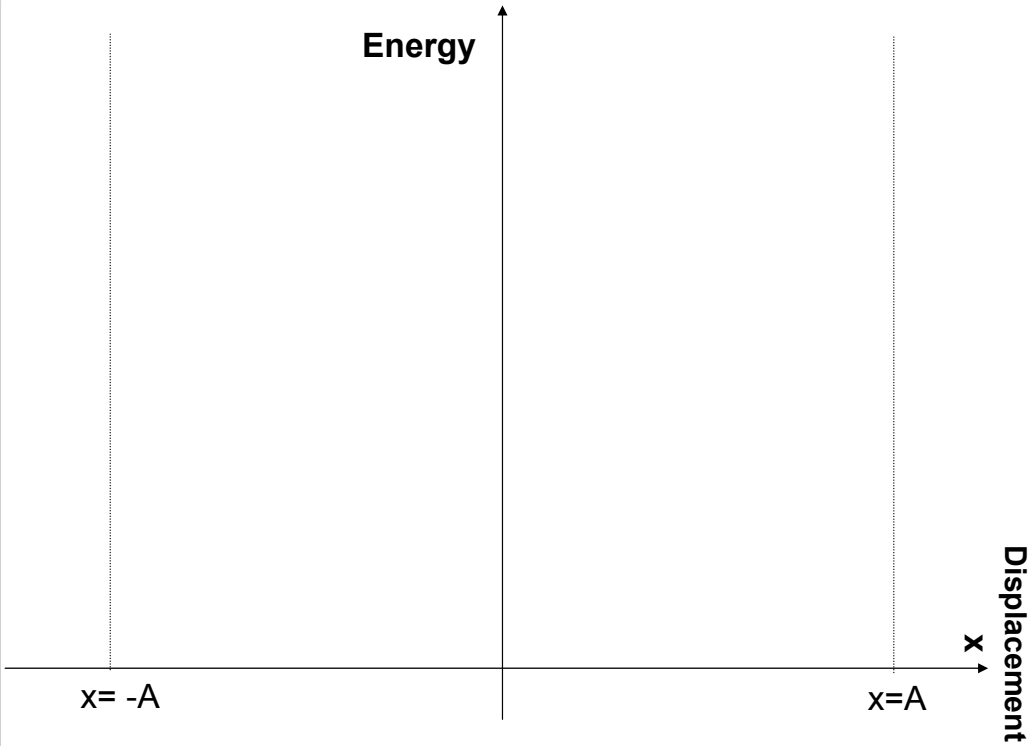
KE is a ……..

……

so graphically ……
has a …….. form.



From this we now construct a more complete SHM energy graph



Question: At what value of *x*, does: KE = PE ?

Summary so far for SHM.

For an undamped spring / mass system;

- Equation of motion; (Derivation, units).
- Solution to equation of motion

  Meaning of amplitude.

  Particular attention to phase ϕ.
- Energy of SHM;

  This is constant.
- Knowledge of graphical form of energy (PE, KE and total E).
- Relation between; x, $\dot{x}$ and $\ddot{x}$

Complex number representation (1)

is a complex number

Where a and b are real numbers, and $j = \sqrt{-1}$

Complex number = (real part) + j .(imag. part)

We can represent a complex number graphically;

Real part =

Imaginary part =

Modulus =

Argument = ϕ =

Complex number representation (2).

There is a remarkable formula in maths;

Using this, we can represent complex numbers with polar notation;

By equating real parts and imaginary parts;

We apply the complex number representation to the analysis of physical phenomena;

eg. …… in periodic motion: …… in circuits.

The reason for this is that the maths of the analysis is much easier.

Forces and voltages are not really complex quantities (physically they only have a real part).

- we represent force as ……
- only the real part has……...

ie. We write _____but ……there is only

Complex number representation (3); example of use...

We will use the fact that;

$$e^{j\phi} = \cos\phi + j.\sin\phi$$

The equation of motion for undamped SHM (spring / mass) is;

As a solution for this, let's try the following;

$\Longrightarrow$

Where A= constant length, and ϕ is a constant.

Complex number representation (4); example of use...

Displacement

Velocity

Acceleration

$\Rightarrow \quad \ddot{x} =$ **It works!**