Looking further at the dispersion relation for beaded string

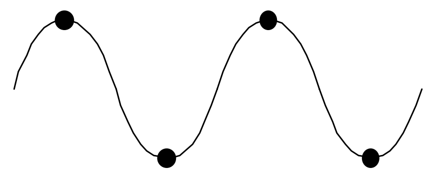
Behaviour at special points on dispersion curve (2)

When ……….. then in this case;

Therefore this represents the situation ………………..
………………………………………………………………..

$$y_n = A\exp[i(\omega t - kna)] \quad = A\exp(i\omega t)\exp(-in\pi)$$

It's a standing wave with ………… at the particle positions, and ………… half-way between.



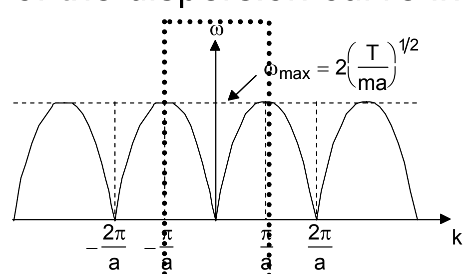
then

Since the group velocity is the velocity ………………………..
…………………………………………………………………..

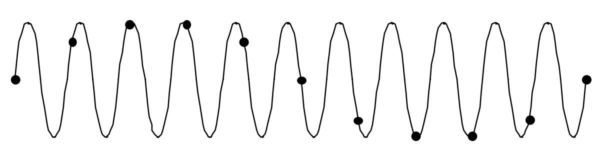
Points to note about the Brillouin zone

The first Brillouin zone

- Remember this is the region of the dispersion curve in the range



- Values of *k* outside this region correspond to ………………………………………………………………..



- Gives exactly same particle motions as ……………… (which is ………………………………………).

Waves on finite periodic structures

Example system.

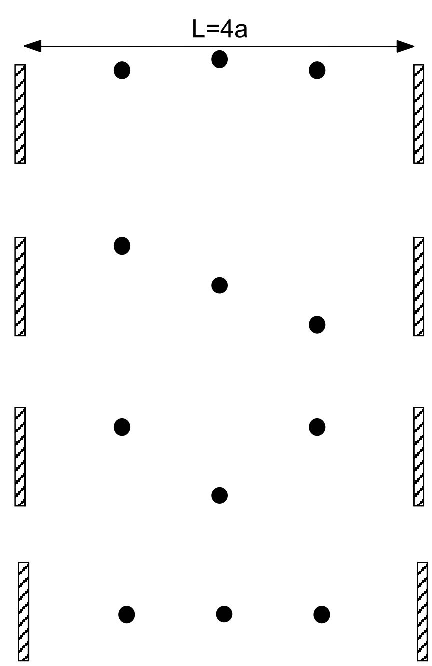
Suppose we have a string of length ……, <u>fixed at both ends</u>, with 3 particles positioned at ……………… along it.

We know that the normal modes of the string on its own are just standing waves with wavelengths
λ = …………………………….. etc,

(i.e. etc) where in this case $L = 4a$.

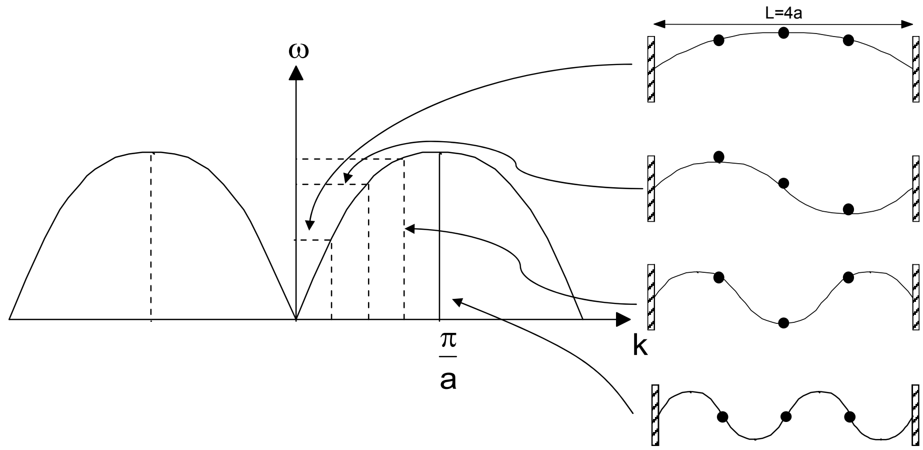
Waves on finite periodic structures (2)

- For the periodic structure, only the modes within the first Brillouin zone, $-\pi/a < k \leq \pi/a$ need be considered:
- These have
- The next mode, $k = \pi/a$, is at the edge of the zone, but all the particles are positioned at ………………..



Specific example of a 3-beaded string

Waves on finite periodic structures (3)

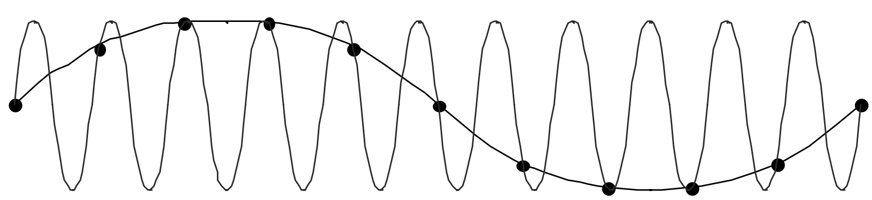


- The modes marked are the ………………………………… – the actual modes are standing waves consisting of equal amplitude travelling waves, travelling in opposite directions.

Brillouin zone explanation

Waves on finite periodic structures (4)

- Values of *k* outside this region correspond to wavelengths less than 2*a*, e.g. red curve:

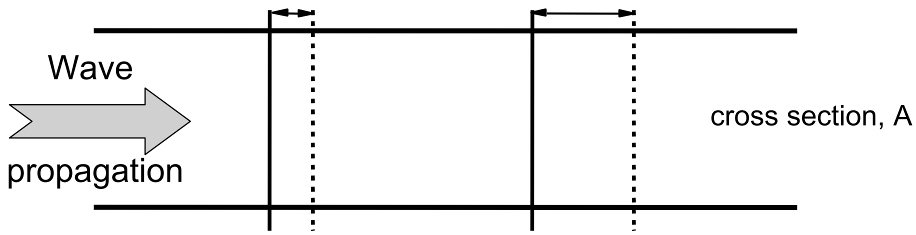


- Gives exactly same ………………… as black curve (which is within the first Brillouin zone).

VIII Other examples of waves

…………….. waves in a bar; e.g. sound waves in a solid.

Diagram shows part of a solid bar:
– material in bar is displaced from equilibrium by wave



- At x_0 the displacement is ξ
- At x_0 + dx the displacement is $\xi + d\xi$ where

Longitudinal waves in a bar (2)

Young's modulus of a material is: ……………………..

- it's a measure of a material's resistance to being stretched
- the wave is stretching the material between x_0 and x_0 + dx by an amount dξ, so;

• Hence the force exerted on the element, by the bar to the left is:

minus because when element is
stretched, force is from right to left

Longitudinal waves in a bar (3)

At the other side of the element, $x_0 + dx$ the force on the element is

(this time if the element is stretched the force is from left to right – positive)

- but is different at the two ends of the element, and
- hence ………………………………………......

Longitudinal waves in a bar (4)

The rate of change of force with distance is;

- so the force at $x_0 + dx$ is;

force at x_0

rate of change × distance from x_0

- Thus the net force on the element dx is;

Longitudinal waves in a bar (5)

$$F = EA\frac{\partial^2 \xi}{\partial x^2}dx$$

- The mass of the element (*dx*) is;

- So Newton's 2nd law for the element of the bar *dx* is;

force = *mass* × *accel'n*

- this is the ……………………………………………….. ……………………