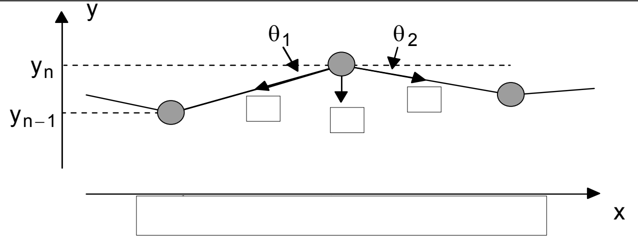
VII Waves on periodic structures

- Consider ……………………………………………………...
……………………………………………………………………….



- We want to investigate the properties of ……………….
……………………………………………………………………
- This is a simplified analogue of a …………………..:
………… regularly ……….. and interacting *via* …………..
……………. (rather than bits of string) – many of the results will be relevant to waves in a crystal.

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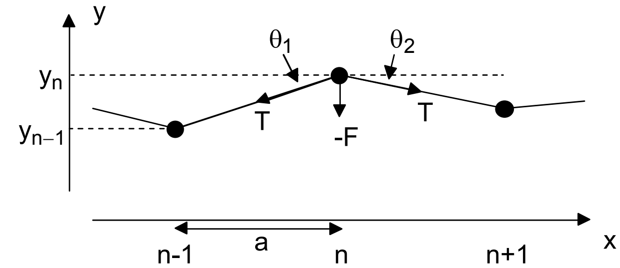
Waves on periodic structures (2)

Setting up the equation of motion:



- Vertical force on ………. particle is

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Waves on periodic structures (3)

Newton's 2nd Law:

Similar to harmonic oscillator: ……………. ∝ ………………

- BUT acceleration of *n*th particle depends on displacement, of that particle AND its two nearest neighbours.
- Coupled harmonic oscillators → WAVES

To solve the equation, we try harmonic wave solution:

Here we have taken the origin of *x* to be at the zeroth particle, so the *n*th particle is at …………. .

Adjacent particles oscillate with the same ……………..,
but are successively shifted in ………… by ……….

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Waves on periodic structures (4)

$y_n = A\exp\left[i(\omega t - kna)\right]$ $\qquad m\dfrac{d^2 y_n}{dt^2} = \dfrac{T}{a}\left(y_{n-1} - 2y_n + y_{n+1}\right)$

Differentiate and substitution back into equation of motion:

Waves on periodic structures (5)

use $\cos\alpha = \left(e^{i\alpha} + e^{-i\alpha}\right)/2$

use $\cos 2x = 1 - 2\sin^2 x$

We now produce the …………………………….. (i.e. ω as a function of *k*).
[From this we can calculate the group and phase velocities].

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Waves on periodic structures (6)

$$\rightarrow \quad \omega = 2\left(\frac{T}{ma}\right)^{1/2}\left|\sin\left(\frac{ka}{2}\right)\right|$$

Calculate the phase velocity:

Calculate the group velocity:

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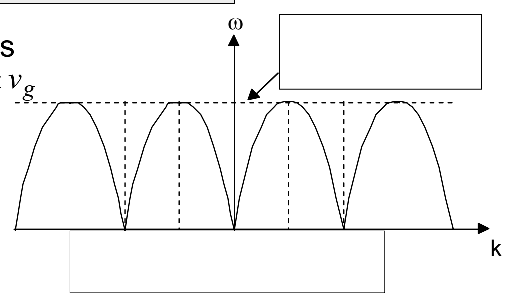
Dispersion relation

The travelling wave has normal dispersion $v_p \geq v_g$

$$\omega = 2\left(\frac{T}{ma}\right)^{1/2}\left|\sin\left(\frac{ka}{2}\right)\right|$$



No travelling wave solution for
– (lattice behaves like ………………)

Periodic in *k* with period $2\pi/a$ – it will turn out that the motions of the particles are fully described by *k*'s in the range $-\pi/a < k \leq \pi/a$
– this is called "the first ……………………." – important in physics of solids

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Behaviour at special points on dispersion curve (1)

$$\omega = 2\left(\frac{T}{ma}\right)^{1/2}\left|\sin\left(\frac{ka}{2}\right)\right|$$

- *k* = 0 at the long wavelength limit
- The sine function becomes linear at low *k* (…………..), so the dispersion relation looks like that of a non-dispersive wave on a string, with

- long wavelength → many particles per wavelength
- like continuous string of linear density $(\rho = m/a)$

$$v_p = \sqrt{\frac{T}{\rho}} = \sqrt{\frac{Ta}{m}}$$

Practice question

Q. Estimate the cut-off frequency for a travelling wave on a beaded string with beads of mass 0.01 kg separated by a distance of 0.05 m, when the tension is 10 N.

Answer: