Transmitted and reflected power

Reminder and definitions of T_p and R_p at a boundary

- Remember, and
- These are the ………………. transmission and reflection coefficients.
- The rate of energy flow in a wave is related to the amplitude by:
- Define POWER transmission and reflection coefficients as:

and

Transmitted and reflected power (2)

Here we calculate the refl. and transm. power in terms of *z*.

$$R_p = \frac{\text{reflected power}}{\text{incident power}}$$

$$T_p = \frac{\text{transmitted power}}{\text{incident power}}$$

- Energy conservation works:
- Note not just ……

Impedance matching

$$T_p = \frac{4 z_1 z_2}{(z_1 + z_2)^2}$$

$$R_p = \frac{(z_1 - z_2)^2}{(z_1 + z_2)^2}$$

Reminder and definitions of T_p and R_p

- The condition for maximum transmitted power is that
  - –This is obvious since this is the case of a single string (no junction) where clearly ……………………..
  - –We could prove this by …………………….. T_p wrt z_2 to find the turning point.

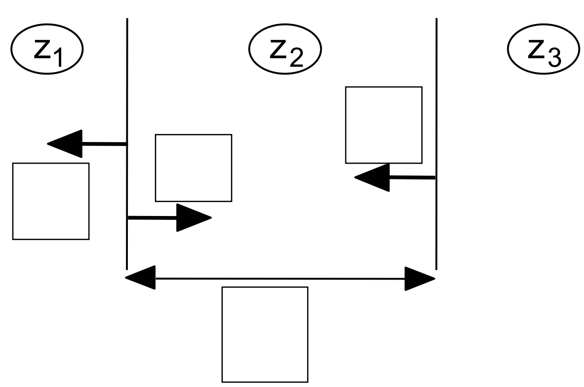
There is a cunning way of getting ……………………….. between two media of ………… *z* …..

i.e. ……………………….

Impedance matching (2)-the quarter-wave transformer

Setting up conditions for impedance matching

- We want a wave of wavelength *l* to pass from a ……………………………………………………………..
- To do this we give the middle medium an impedance of with a ……………………….. between them.



Remember;

The quarter-wave transformer (2)

Setting up conditions for imp. matching

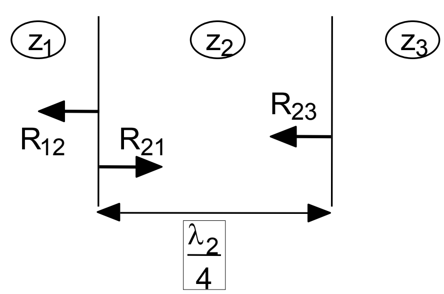
- The trick is going to be to make the reflections from the ………………………………… …………………and hence to cancel one another perfectly.
- The width of .... ensures that the two reflections are …………………………….

The quarter-wave transformer (3)

Putting together equations to prove imp. matching

$$R_{ij} = \frac{z_i - z_j}{z_i + z_j}$$

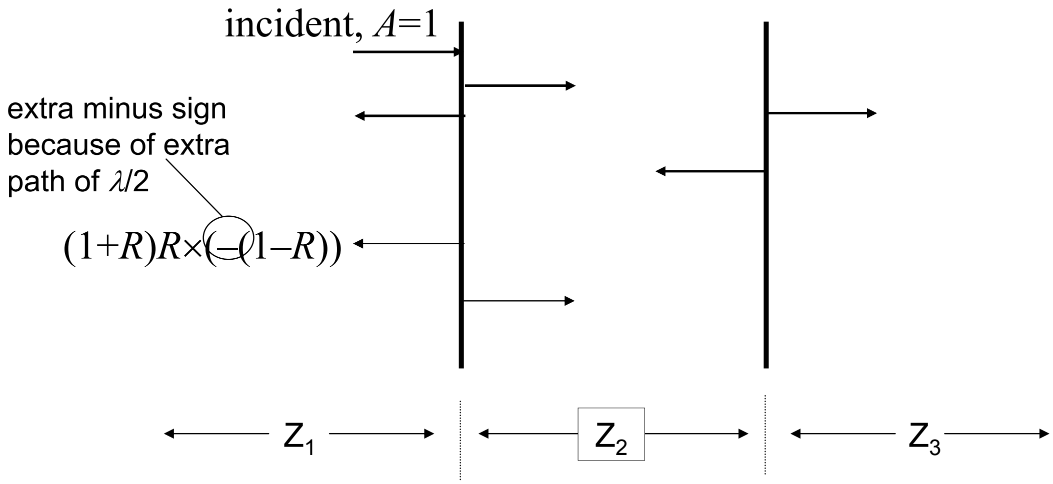
- From z_1, z_2 and z_3 we can calculate the various reflection and transmission coefficients:



The quarter-wave transformer (4)

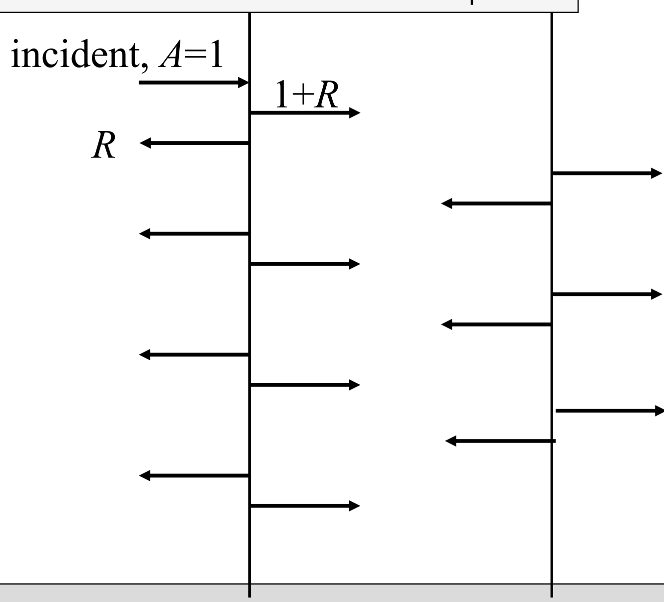
Deriving the quarter wave relations for the *series* equation

Amplitude R and T coefficients for each interface:



The quarter-wave transformer (5)

Deriving the quarter wave relations for the *series* equation



Writing in terms of a series…

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The quarter-wave transformer (6)

Deriving the quarter wave relations for the *series* equation

$$R_{\text{total}} = R - \left(1 - R^2\right)R\left(1 + R^2 + R^4 + ...\right)$$

$$T_{\text{total}} = (1 + R)^2\left(1 + R^2 + R^4 + ...\right)$$

From maths, we know that the sum of a geometric series,

So, identifying *x* with R^2 (which is less than 1), what are the values for R_{total} and T_{total}?

and

i.e. all incident energy is ……………………………………

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The quarter-wave transformer (7)

Summarising the quarter wave relations

- this confirms that there is no reflected power:

$$R_p = R_{\text{total}}{}^2 = 0$$

- the first reflection ……………………………………… ……………………………………………………
- The width of ………ensures that the subsequent reflections are all out of phase with the first.
- The value of z_2 is chosen to make the cancellation complete. $z_2 =$ ………..
- The value of T_p should be 100% (i.e. = 1).

We'll prove this now…

The quarter-wave transformer (8)

Proving that T_p is 100% in quarter-wave transformer

Transmitted power is given by:

$$T_p = \frac{z_3}{z_1} T_{\text{total}}^2 = \frac{z_3}{z_1}\left(\frac{1+R}{1-R}\right)^2 = \frac{z_3}{z_1}\left(\frac{1+\frac{z_1-z_2}{z_1+z_2}}{1-\frac{z_1-z_2}{z_1+z_2}}\right)^2$$

$$= \frac{z_3}{z_1}\frac{z_1^2}{z_1 z_3} = 1$$

(remember $z_2 = \sqrt{z_1 z_3}$

In other words: ……………………………..

ENERGY IS ……………………..: i.e. even in this impedance matching system,

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PHY 1106: Waves and Oscillators (Lecture 19)