(VI) Reflection and Transmission of Waves

Characteristic Impedance

This is a property of the medium in which the wave propagates, which describes how hard it is to set up a wave in the medium.

For all mechanical waves this takes the form;

i.e.

UNIVERSITY of EXETER
Dr. Pete Vukusic, Exeter University:
PHY 1106: Waves and Oscillators (Lecture 18)

Characteristic impedance (2)

e.g. for transverse waves on a string;

Consider the wave: $y = y_0 \exp[i(\omega t - kx)]$

$$z = \frac{\text{transverse driving force}}{\text{transverse velocity}} = \frac{-T\frac{\partial y}{\partial x}}{\frac{\partial y}{\partial t}}$$

$$= \frac{T}{\omega / k}$$

since the phase velocity $v_p = \sqrt{\frac{T}{\rho}}$

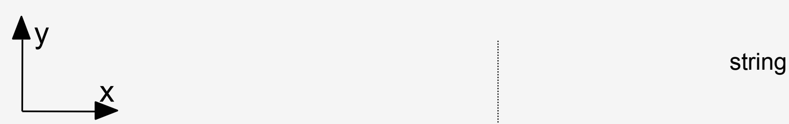
Characteristic impedance (3)

Writing energy flow in terms of *z:*

$$\text{Rate of energy flow} = \frac{1}{2} T\omega k {y_0}^2$$

Reflection and Transmission at a boundary

Consider this string that has a density junction in it at x = 0



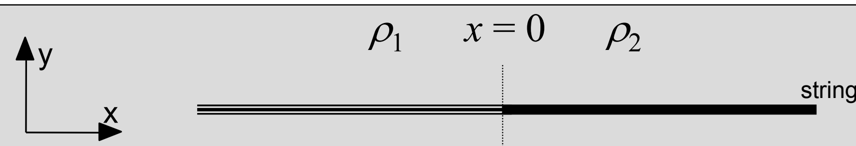
(Tension in this string is the same everywhere)

(*z* turns out to come in useful)

The density junction affects the travelling wave in the following way...

Reflection and Transmission (2)

Here we calculate the R and T coefficients.



Incident: $\rightarrow$

$\rightarrow$ Transmitted:

Reflected: $\leftarrow$

Amplitude Reflection Coefficient:

Amplitude Transmission Coefficient:

Reflection and Transmission (3)

Here we begin to work out the R and T in terms of z.

Use the following boundary conditions at x = 0:

1. ……………………………………………………… – otherwise the string would break

2. The ……………………………………………… must be ………………...
   - Remember the transverse force is proportional to this derivative. $F = -T\frac{\partial y}{\partial x}$
   - If the force were not continuous, there would be a finite net force on an infinitesimal segment of the string at *x* = 0, which would cause an infinite acceleration.

Reflection and Transmission (4)

Here we're still working out the R and T in terms of z...

Using condition 1 at x = 0:

$$A_1 \exp(i\omega_1 t) + B\exp(i\omega_1 t) = A_2 \exp(i\omega_2 t)$$

This can only be true for all times if $\omega_1 = \omega_2 = \omega$, in which case:

Reflection and Transmission (5)

Here we're still working out the R and T in terms of z…

Using condition 2: $\frac{\partial y_i}{\partial x} + \frac{\partial y_r}{\partial x} = \frac{\partial y_t}{\partial x}$ at x = 0, at all times:

Now remember that $z = \frac{Tk}{\omega}$ so for fixed ω and fixed T, then z is proportional to k

Thus the second boundary condition becomes:

UNIVERSITY of EXETER

Reflection and Transmission (6)

Here we calculate the R in terms of *z*.

$$A_1 + B = A_2 \quad (A_1 - B)z_1 = A_2 z_2$$

Eliminate A_2 by dividing these two equations:

$\longrightarrow$

Dr. Pete Vukusic, Exeter University:
PHY 1106: Waves and Oscillators (Lecture 18)

UNIVERSITY of EXETER

Reflection and Transmission (7)

Here we calculate the T in terms of *z*.

$$R = \frac{(z_1 - z_2)}{(z_1 + z_2)}$$

Now from the first condition,

$$A_1 + B = A_2$$

Therefore

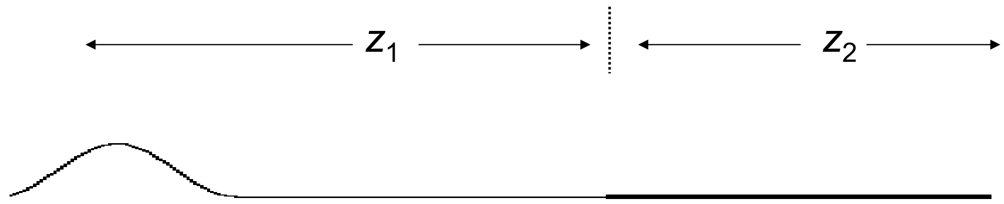
Hence

Dr. Pete Vukusic, Exeter University:
PHY 1106: Waves and Oscillators (Lecture 18)

Reflection and Transmission (8)

$$R = \frac{(z_1 - z_2)}{(z_1 + z_2)} \qquad T = \frac{2z_1}{(z_1 + z_2)}$$

- Wave goes from less dense to more dense region:



R is NEGATIVE:

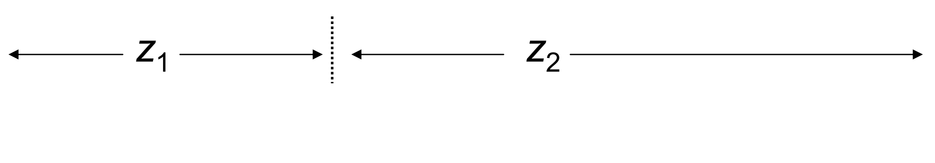
transmitted amplitude < incident amplitude ($T < 1$)

Dr. Pete Vukusic, Exeter University:
PHY 1106: Waves and Oscillators (Lecture 18)

Reflection and Transmission (9)

$$R = \frac{(z_1 - z_2)}{(z_1 + z_2)} \qquad T = \frac{2z_1}{(z_1 + z_2)}$$

- Wave goes from more dense to less dense region:



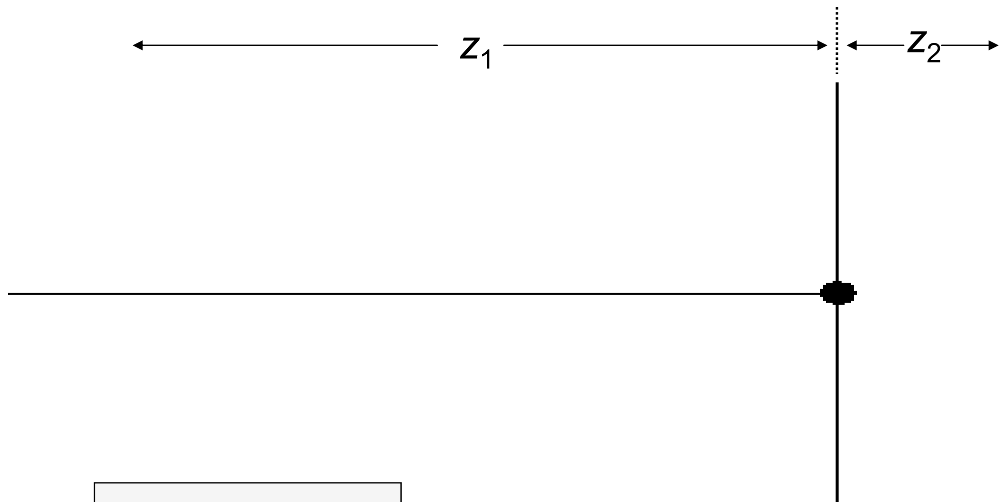
R is POSITIVE

transmitted amplitude > incident amplitude ($T > 1$)

Reflection and Transmission (10)

$$R = \frac{(z_1 - z_2)}{(z_1 + z_2)} \quad T = \frac{2z_1}{(z_1 + z_2)}$$

- String fixed rigidly at x = 0:



Reflection and Transmission (11)

$$R = \frac{(z_1 - z_2)}{(z_1 + z_2)} \quad T = \frac{2z_1}{(z_1 + z_2)}$$

- String has free end at x = 0: