Group velocity and dispersion

$$v_g = \frac{d\omega}{dk}$$

Phase velocity and group velocity are equal when the …………………………………………………………..

i.e. non-dispersive waves (all the examples we have seen so far)

However this is not generally the case:

(i.e. phase velocity is a function of k)

→ …………………

Two types of dispersion

Using $k = \frac{2\pi}{\lambda}$ and the chain rule:

→ group velocity and phase velocity NOT equal

Two types of dispersion (2)

1. When ……………………………………...

‘………. DISPERSION’ $v_g < v_p$

2. When …………………………………….

‘……………... DISPERSION’ $v_g > v_p$

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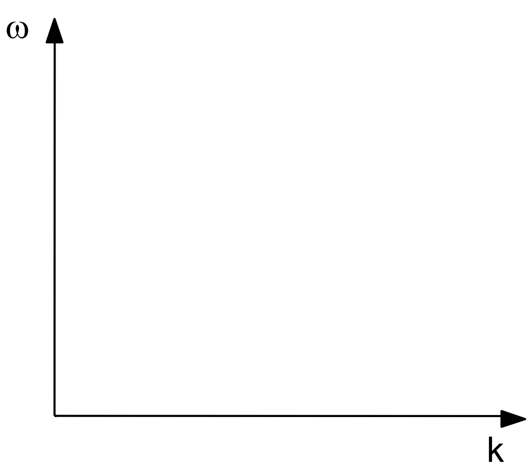
Two types of dispersion (3)

Dispersion relation (relation between ω and k) has same role as energy-momentum relationship for particles:



Normal dispersion

………………………………………………………………..

This creates normal dispersion.

(See animation 1)

Anomalous dispersion

……………………………………………………………….

This creates anomalous dispersion.

(See animation 2)

Dispersed wave on a string

The equation of motion for an element of a string was:

Add damping force, proportional to velocity

The wave equation becomes:

Dispersed wave on a string (2)

$$\rho \frac{\partial^2 y}{\partial t^2} = T\frac{\partial^2 y}{\partial x^2} - b\frac{\partial y}{\partial t}$$

Try a solution of the form $y(x,t) = y_0 \exp[i(\omega t - \gamma x)]$

Differentiate and substitute

Dispersed wave on a string (3)

So to satisfy the equation of motion, γ has to be complex. i.e.

The solution becomes

k_1 plays the same role as *k* in the un-damped case, with $v_p = \dfrac{\omega}{k_1}$

k_2 is the rate of exponential decay with distance *x*.

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Dispersed wave on a string (4)

It is straightforward to find the values of k_1 and k_2

(see notes on web for derivation (not examinable))

The results are:

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Dispersed wave on a string (5)

$$k_1 = \omega\left[\frac{\rho}{2T}\left(\sqrt{1+\frac{b^2}{\omega^2\rho^2}}+1\right)\right]^{1/2} \qquad k_2 = \omega\left[\frac{\rho}{2T}\left(\sqrt{1+\frac{b^2}{\omega^2\rho^2}}-1\right)\right]^{1/2}$$

- When b = 0,

- When b ≠ 0,

$v_g > v_p \rightarrow$ ANOMALOUS DISPERSION (typical of damped systems)

- the rate of damping $(\propto k_2)$ increases as *b* increases

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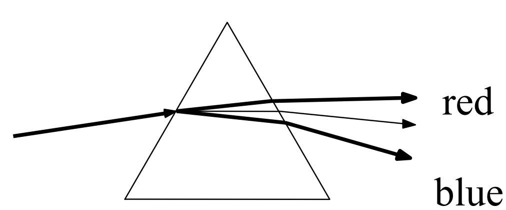
Final comments on dispersion (1)

- Anomalous dispersion is typical of systems where there is ……………………...

- Normal dispersion; e.g. for light travelling in dielectric media (i.e., glass) this is the reason for the word ‘normal’. It causes white light passing through a prism to be ……………………………………. (‘disperses’ it)....

Snell’s law: amount that light bends is proportional to refractive index, *n*, and $n = \frac{c}{v}$

Blue light travels slower than red (normal dispersion) so bends more.



Final comments on dispersion (2)

Wave packets traveling in dispersive media generally change shape as they propagate.

Components of ……………………….. (i.e. wavelength) travel at different speeds.

e.g. ……………………………………………………….
………………………………………………………….

Final comments on dispersion (3)

• When the group and phase velocities of a wave packet are different, …………………………………………….

two kinds of dispersion – ……………….
– …………………..

• Dispersive waves on a string caused by ……………., leads to …………………. dispersion.

• Light travelling through glass displays normal dispersion, → ……………………………………….