Partial standing waves

Standing waves are important because they occur in nature when ………………………………………………………
……………………...

If the reflection is not perfect, the ……………. of the reflected wave is reduced.

$\rightarrow$ …..……………………………………………..

with R < 1

1-direction

Reflection from “wall” then back in other direction

Two ways of interpreting this equation...

Partial standing waves (2): interpretation 1

1st term — 2nd term

$$A_{tot}(x,t) = A\sin(\omega t - kx) + AR\sin(\omega t + kx)$$

(split just the 1st term)

(then add 2nd term and trig. simplify)

………. *wave* ………….. *wave*

A …………………………… is the superposition of a travelling and a standing wave.

(Interpretation 1)

Partial standing waves (2): interpretation 2

$$A_{tot}(x,t) = A\sin(\omega t - kx) + AR\sin(\omega t + kx)$$

Use trig formulae:

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$
$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

“A” = ωt, “B” = kx

A partial standing wave is the superposition of two standing waves (different amplitudes and phases)

(Interpretation 2)

Partial standing waves (4)

Interpretation 1 is more intuitive:

A standing wave has nodes where the amplitude is zero.

A travelling wave has the ……………….. everywhere.

Interpretation 1 $\rightarrow$ a partial standing wave will behave somewhere in between:

it will have ……………………………..
…………………….., but the minima will not be zero.

Partial standing waves (5)

$$A_{\text{tot}}(x,t) = A\big((1+R)\sin(\omega t)\cos(kx) - (1-R)\cos(\omega t)\sin(kx)\big)$$

Rewrite this as; $a\cos(\omega t) + b\sin(\omega t)$ (where *a* and *b* are functions of *x*)

We need to derive an expression for c in: $c\cos(\omega t + \varphi)$

To do this, use $\cos(A+B) = \cos A\cos B - \sin A\sin B$

to expand $c\cos(\omega t + \varphi)$

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Partial standing waves (6)

The expression for *c* in: $c\cos(\omega t + \varphi)$

c it is the position-dependent amplitude of the partial standing wave:

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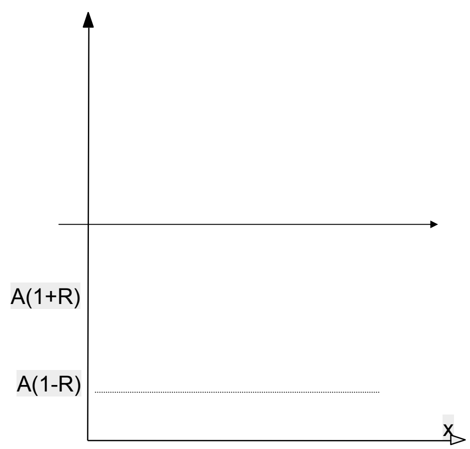
Partial standing waves (7)

The graphs show the variation of amplitude (max disturbance) with distance for:

(a) a standing wave (*R*=1)

(b) and for a partial standing wave with an amplitude reflection coefficient R (≠1).



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Partial standing waves (8)

The ratio of the maximum to minimum amplitude is called the ……………………… (SWR) and gives the value of *R*:

Wave packets

A wave will only be totally 'pure' (single frequency) if it persists from $t=-\infty$ to $t=+\infty$

In real situations, waves travel as packets – superpositions of waves of different ω and k.

e.g. consider the superposition of
and

Wave packets (2)

$$A_1 = A_0\cos(\omega t - kx)$$
$$A_2 = A_0\cos[(\omega+\Delta\omega)t-(k+\Delta k)x]$$

slowly varying envelope **close to original ω and k**

where we have used the trig identity

$$\cos A + \cos B = 2\cos\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)$$

Wave packets (3)

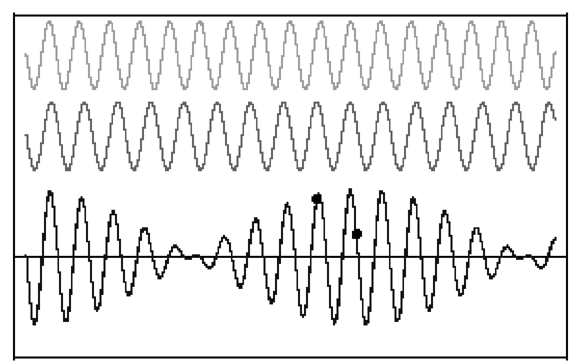
$$A_1 + A_2 = 2A_0\cos\left[\frac{\Delta\omega}{2}t-\frac{\Delta k}{2}x\right]\cos\left[\left(\omega+\frac{\Delta\omega}{2}\right)t-\left(k+\frac{\Delta k}{2}\right)x\right]$$

Graph of this function reveals the existence of "………………."

$2A_0$

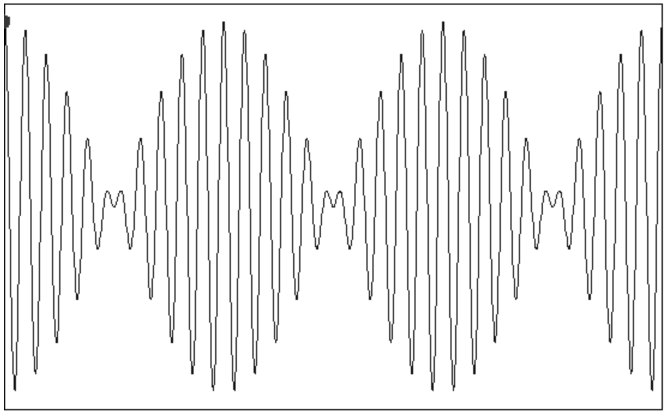
Wave packets (4)

Two travelling waves with different ω and k *can superpose to produce travelling wavepackets*.



Wave packets (5)

Red dot indicates progress of wave packet



Velocity of dot is the …………………. (i.e. the velocity of wavelets)

Here:

See web for other anims.

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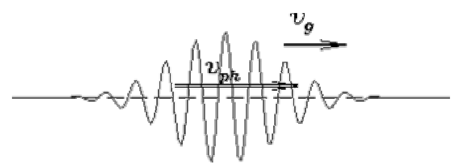
Group velocity and phase velocity (1)

There is now something else to consider, in addition to the velocity of the wavelets;

The **……………………** is the 'phase velocity' of the envelope (= velocity of the packet [group]).



The "wave packet" moves at V_g and "wavelets" move through it at V_{ph}

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Group velocity and phase velocity (2)

Comments:

- Group velocity is the velocity …………………… …………………………..
  – therefore important!
- in general, …………………………………
- The **<u>dispersion relation</u>** (ω vs. k) allows us to calculate the group velocity (see later).

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Group velocity and phase velocity (3)

- Phase velocity is the velocity of points of constant phase angle in a wave of the form

$$A(x,t) = A_0 \sin(\underbrace{\omega t - kx}_{\text{phase}})$$

- Group velocity is the velocity of points of constant amplitude in a group of waves.

Example question

A wave on a piano string obeys the dispersion relation $\omega = ck + dk^3$ with c and d being positive constants.

Write expressions for the phase and group velocity of the wave.