(V) Superposition of Waves

• If two waves are present in a medium, then their …………… (e.g. displacement of string) simply ……..

• …………………………………… – comes from the fact that the wave equation is linear in $A(x,t)$.

• If the solutions of the wave equation are;

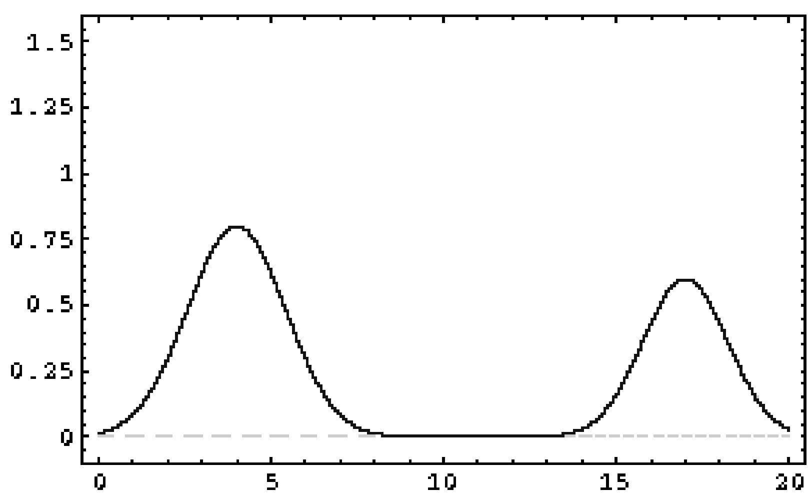
$$A_1(x,t) \text{ and } A_2(x,t)$$

then so is

Superposition example

• Two pulses approach one another on a string

• Their amplitudes …………………………………….



• they ………………………………………………..

Wave superposition analysis

Consider superposing two waves with the same *k*,
...which are moving in opposite directions:

$$A_1(x,t) = A_{10}\sin(\omega t - kx)$$ and $$A_2(x,t) = A_{20}\sin(\omega t + kx)$$

Adding them together gives

<u>CASE 1</u>: **equal amplitudes** ($A_{10} = A_{20}$).

Use the trigonometric identities

$$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$

and $$\sin(\theta-\phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$$

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Superposition of 2 waves with same *k*

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PHY 1106: Waves and Oscillators (Lecture 15)

Substituting using trig. identities: (and $A_{10} = A_{20} = A_0$).

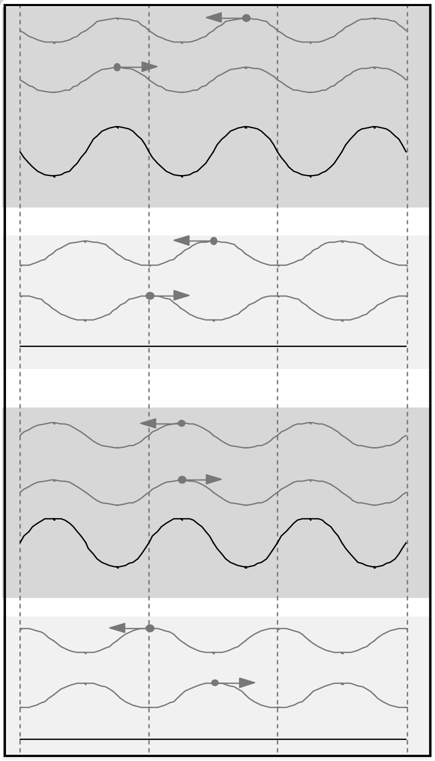
$A_{tot}(x,t) =$

- This is a ……………………….
- Its amplitude is… $2A_0 \sin \omega t$
- i.e. it changes with time

A_{tot} changes with position

amplitude changes with time

Standing wave picture in space



At this time the two waves …….
……… → ADD to a maximum

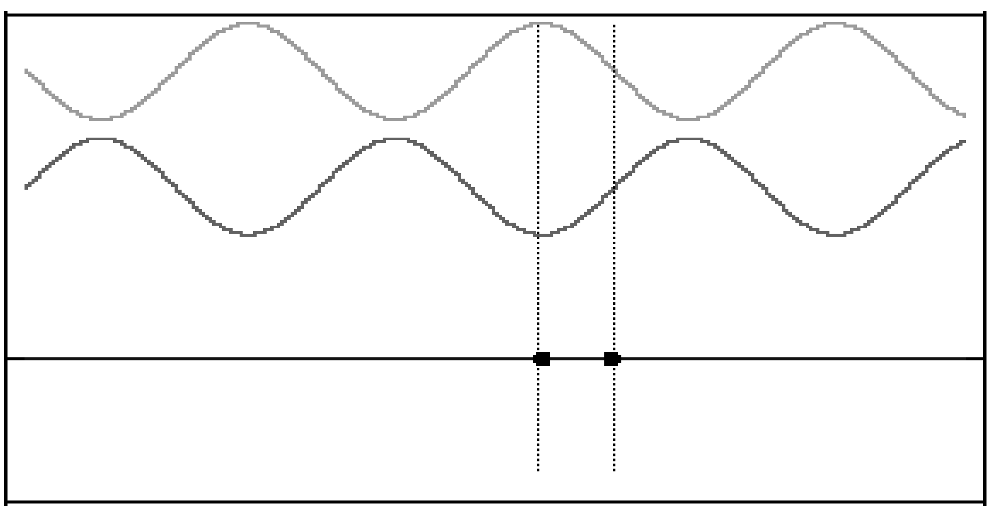
Now they have each moved by $\lambda/4$ in opposite directions
……………………………………

Now they are ……………………

And out of phase...

Superposition animation

The grey traces (top) are the two travelling waves,
– blue (bottom) shows the ……………..



Black dots show …………………………………………….

Comments

1. Some points in space have no disturbance, at any time:

$= 0$ when

These points are ....

2. Some points have a maximum disturbance equal to the sum of the amplitudes:

when

These points are ....

Energy transfer in a standing wave

For our standing wave $y \equiv A_{\text{tot}}(x,t) = 2A\sin\omega t\cos kx$

Remember for a string, the rate of energy flow was;

(So by differentiation and substitution);

$$P(x,t) = T\omega k(2A)^2 \sin(\omega t)\sin(kx)\cos(\omega t)\cos(kx)$$

Energy transfer in a standing wave (2)

Now compare

with the result for a travelling wave

Remember the ……. function has a time average of ……

What is the time average of the function $\sin(2\omega t)$?

→ ………..

In other words;

……………………………………………………………

(the energy carried by the two travelling waves cancels)

Normal modes of a stretched string

This analysis is for vibrations of a string with fixed ends.
Since the ends are fixed, the '*boundary conditions*' for this problem are that the wave displacement be zero at $x = 0$ and $x = L$ (the string's length)

So; NODES at $x = 0$ and L, imply STANDING WAVES

therefore, the equation for the wave is something like this:

But with the node at $x = 0$, then the solution only uses $\sin(kx)$

Normal modes of a stretched string (2)

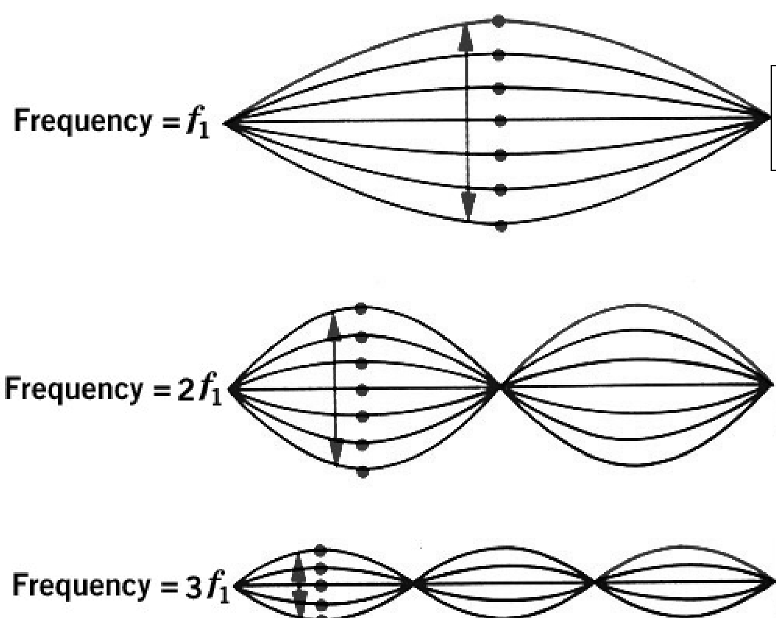
To satisfy the boundary condition at $x = L$, we find that k can only take certain values:

i.e. since then

where n is a positive integer.

This corresponds to

An integer number of half-wavelengths have to fit into the distance L



Normal modes of a stretched string (3)

Taking the *sin*(ωt) term (we could use that or *cos*), we have the general equation of the standing wave;

where $\omega_n = v k_n = \frac{n\pi}{L}\sqrt{\frac{T}{\rho}}$

These functions are called the ………………………, or ……………………., of the system.

(they are the only allowed vibrational modes)

Normal modes of a stretched string (4)

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Significance:
If the string is set to oscillate in one of its *normal modes*, it will continue to do so for ever (ignoring dissipation)
 - ………………………………………………………….

Superposition:

- ……………………………………………………….. ………………………………….
- this is a general property of eigenfunctions – in the case of *sines* and *cosines*, it is called a ………………………. (or harmonic series)

$$\Psi_{\text{general}} = \sum_{n=1}^{\infty} c_n \Psi_n$$

- the time dependence of each component in the sum is different, so the general wave will not retain its shape as time progresses.