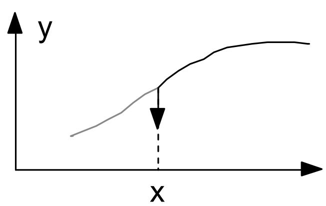
Energy transfer in waves on a string (1)

Dr. Pete Vukusic, Exeter University:
PHY 1106: Waves and Oscillators (Lecture 14)

One of the defining properties of a wave …………………. ……………………………………..

But how does it do this ……………………………………?

Consider the harmonic wave $y(x,t) = y_0 \sin(\omega t - kx)$ on a string:



The string to the left of the point x exerts a vertical force, on the part to the right.

That part moves through a distance (in the y direction).

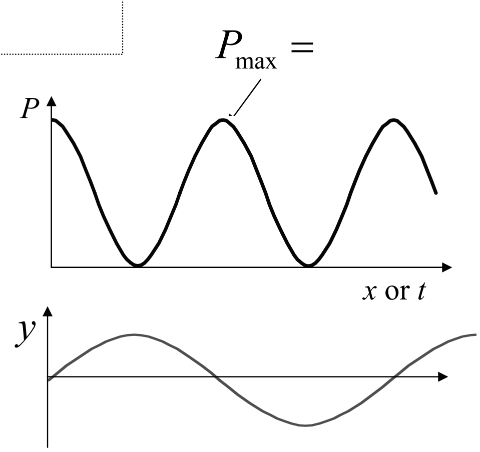
Energy transfer in waves on a string (2)

Dr. Pete Vukusic, Exeter University:
PHY 1106: Waves and Oscillators (Lecture 14)

So, since

$y(x,t) = y_0 \sin(\omega t - kx)$

then



Energy transfer in waves on a string (3)

Points to note:

$$P(x,t) = T\omega k y_0^{\,2}\cos^2(\omega t - kx)$$

- This is the ……………………………… at (x,t).
- Notice that provided k is positive, P is never negative
- ……………………………………………………………

The time-average of P, is

(because the time average of $\cos^2$ is 1/2)

Energy density

Energy density is the total energy per unit length

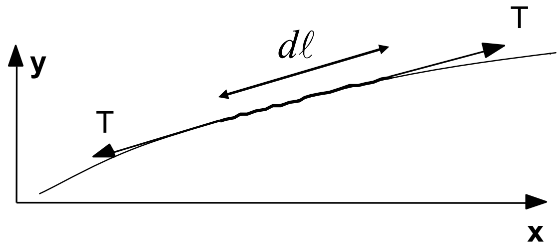
It is the ……………………………………………..

KE per unit length =

Mass per unit length (particle velocity)^2

PE comes from extra stretching of the string due to wave:

Energy density (2)



The extra length due to wave is $d\ell - dx$

where

using the Taylor expansion

and retaining only the first two terms.

Energy density (3)

Thus the PE of the segment

and the PE per unit length

Energy density =

Say; for the harmonic wave; $y(x,t) = y_0 \sin(\omega t - kx)$

so differentiate wrt. *t;*

and wrt *x*;

Energy density (4)

$$\frac{\partial y}{\partial t} = y_0 \omega \cos(\omega t - kx) \quad \text{and} \quad \frac{\partial y}{\partial x} = -y_0 k \cos(\omega t - kx)$$

By substitution:

Energy density = (KE + PE)

But

Energy density =

Energy density (5)

Compare these two energy expressions:

Rate of energy propagation

Different by factor

Energy density E_D =

We can therefore say:

Rate of energy propagation =

– a general result for waves

Example: 2000 Q4

4. A wave on a stretched string $y = a \sin(wt - kx)$ is produced by a generator with frequency 10 Hz and it propagates with a phase velocity of 10 m s^{-1}. Its amplitude is a =0.1 m.

a) Find the velocity of the point at x = 1 m on the string at t = 10 s. [5]

b) Find the average energy transfer rate along the string if its tension is T = 10 N. [5]

(V) Superposition of Waves

• If two waves are present in a medium, ………………….
…………………………………………………………………

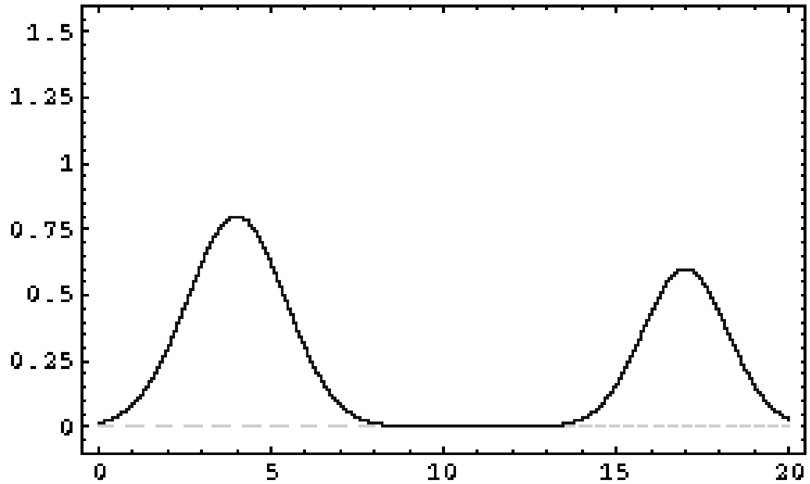
• PRINCIPLE OF …………………….. – comes from the fact that the wave equation is linear in $A(x,t)$.

• If the solutions of the wave equation are;
$A_1(x,t)$ and $A_2(x,t)$

then so is

Superposition example

- Two pulses approach one another on a string
- Their amplitudes ……………………………………



- ……………………………………………………….