The Wave Equation

For oscillators you found that the equation of motion was of the form:

Waves are coupled oscillators, so we expect a …………………………………………………………….
i.e. a ……………………………………………..

In fact we will see that many kinds of wave satisfy the equation:

$$\nabla^2 A = \frac{1}{v^2}\frac{\partial^2 A}{\partial t^2}$$

in 3 dimensions

v is the phase velocity

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Solutions to the wave equation

Any function $A(x,t) = f(vt \pm x)$ is a solution:

Proof:

Let , say. Now, calculate the derivatives:

WRT *x*:

and WRT *t*:

$$\frac{\partial^2 A}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 A}{\partial t^2}$$

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Comments on these solutions

The function *f* can be anything, as long as *x* and *t* appear in it as $vt \pm x$

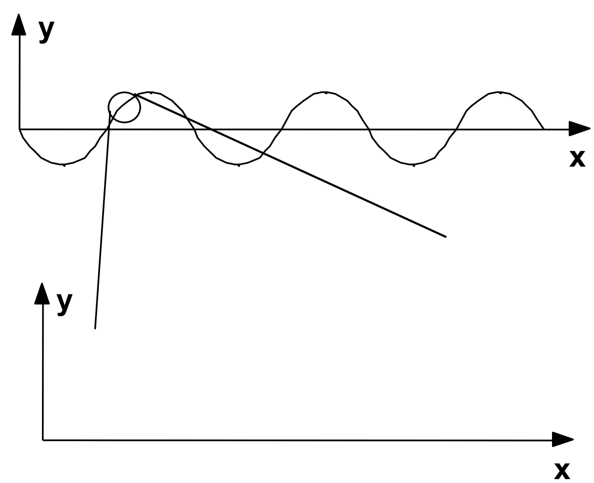
for example, the following harmonic solutions

with are special cases from which we see that *v* is the ……………………..

Waves on a string

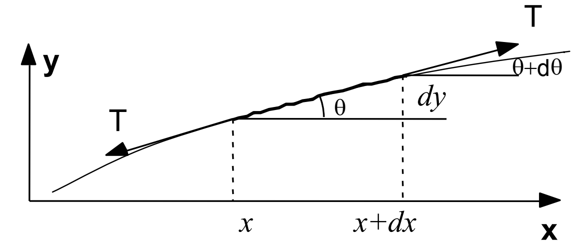
Example of the wave equation

- This shows a snapshot of a transverse wave on a string at some time *t*.
- This one shows the forces on a v. small element of the string.



- ……………………………………
- oscillations are small
- gravity is neglected
- ……………………………..

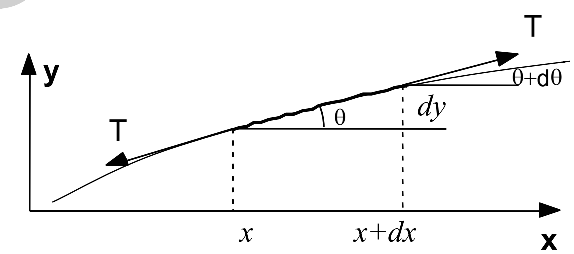
Waves on a string (3)



Length of string between x and $x+dx$ is

– So the mass of a single string element is…….

Waves on a string (4)



The net force on the element, in the y-direction is;

(for small angles, $\sin\theta \approx \tan\theta \left(= \frac{dy}{dx} \right)$)

Waves on a string (5)

Knowing the FORCE and the MASS,

The equation of motion is:

(partial derivatives because *y* is a function of both *x* and *t*, and the force we calculated was at some fixed time *t*)

Hence

Waves on a string (6)

Compare the equation for a wave on a stretched string:

to the general wave equation:

The phase velocity for a wave on a stretched string is;

Briefly:

Note the net force on the element in the *x* direction is zero for small oscillations: it is

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Waves on a string (7)

The wave equation is <u>typical</u> for …………….
……………..

The phase velocity is always of the form

$$\left(= \sqrt{\frac{T}{\rho}} \;\; \textit{for the string} \right)$$

Example: 1998 Q3

A transverse wave on a string with linear density r = 0.1 kg m^{-1} is given by

y = (0.001 m) cos [(15 s^{-1}) t + (7.5 m^{-1}) x]

where m and s indicate the units of the numerical quantities.

For this wave calculate:

(a) the frequency [2]

(b) the wavelength [2]

(c) the phase velocity [2]

(d) the largest transverse speed reached by each point on the string [2]

(e) the tension of the string T [2]