

RMS Voltage

For a driving voltage of the form

or a current of the form

The expression for the RMS (root mean square) value is;

RMS tells us the average of V^2 (or I^2) over a full cycle.

The mains voltage is usually quoted as
But V_0 is higher, and is

Power in AC circuits

In a **mechanical system**,
average power was;

We could derive the expression for power in AC
circuits in exactly the same way.

But we could also write

And also

Resonance in AC circuits.

We have $I = \frac{V}{Z}$

When will the amplitude of the current be maximum? i.e. When is resonance?

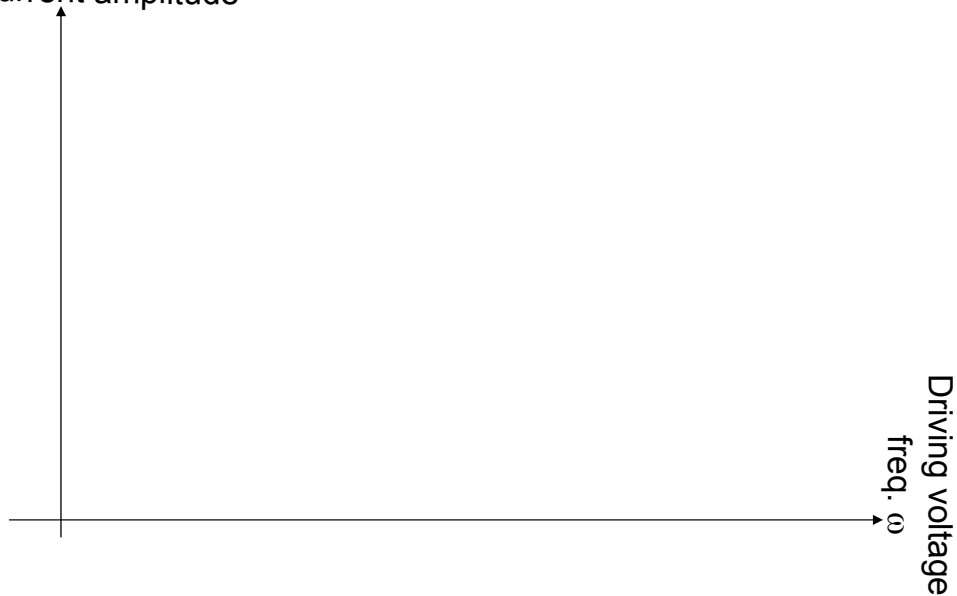
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So, if;

Then Z will be minimum when;

We call this the resonant frequency

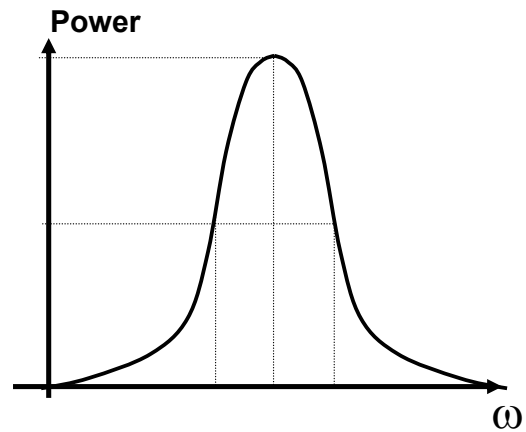
Current amplitude



The current is since Z is a minimum (and equal to R)

Q-factor

The Q-value tells us how sharp the resonance is.



Where for, ω_1 , ω_2 ;

For series LCR circuit;

(Without the long derivation)

Compare to

$Q =$

Practice Question.

Write down the expression for the impedance of a system comprising a resistor in series with a capacitor and inductor.

If $R=1.5\text{k}\Omega$, $L=6\text{H}$ and $C=5\mu\text{F}$, determine the value of the modulus of the impedance when driven at 50Hz.

What is the phase angle between the current and voltage?

Answer.

Modulus of Z;

Phase angle ϕ

Questions

1. In AC theory, describe what is meant by the statement “the voltage across an inductor leads the current by 90 degrees”?
2. With this in mind, go on to describe the phase differences between the voltages across the resistive, inductive and capacitive components within a series LCR circuit.
3. Show that the magnitude of the impedance is $|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$
4. Write an expression for the phase angle between the driving voltage and current in the circuit.
5. If a 2mF capacitor is used in conjunction with a 20Ω resistor, show that a 5mH inductor will cause the system to resonate at a driving frequency of approx.. 50 Hz.
6. With the 5mH inductor in place, calculate the resulting current amplitude I_0 for a driving voltage of the form $V=100\cos 50t$
7. Write down the expression for the average power transferred.
8. Calculate P_{av} at 50Hz for the component values given.

Answer 1. The statement describes the phase between V and I.

The I-V relation is :

The “j” term represents a 90 degree phase relation between the voltage and current phasors. (Sketch diagram).

Answer 2. Describe other two I-V relations.

Highlight presence/absence of j-operator.

Sketch full phasor diagram.

Answer 3. Write / derive impedances of components.

Impedances in series “add”.

Simplify expression.

Obtain magnitude.

Answer 4. Back to expression for Z.

Real and imag. components on Argand diagram.

Trig. produces phase angle expression.

Answer 5. System resonates when current is maximum value. (i.e. $I=V/Z$)

This occurs at minimum Z.

Z is a minimum when inductive and capacitive reactances cancel.

i.e. This occurs when

Then use $\omega = 2\pi f$ for $f = \dots\dots\dots$ Hz.

Answer 6. If $V = 100 \cos 50t$; this means $V_0 = 100$ and $\omega = 50$.

Use $I_0 = V_0 / |Z|$.

But at 50Hz, the system is approx. at resonance,

therefore $|Z| = R$

and

Answer 7.

Answer 8.

Again using $|Z| = R$ at 50Hz, because the system is at resonance,

Also at resonance,

Therefore P_{av} simplifies to

End of section III