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Comments on Longitudinal waves in a bar

Longitudinal waves in a solid bar are analogous to waves on a string:

Sound waves in a solid | Transverse waves on a string

In both cases the phase velocity:

– is larger in a medium ………………………………….

– is smaller in a medium ………………………………..

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Characteristic impedance

Remember that *z* is of the form;

$$z = \frac{\text{driving force}}{\text{appropriate particle/medium velocity}}$$

For a bar of unit cross-sectional area,

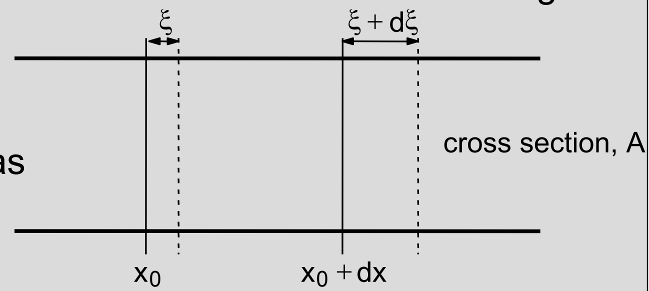
So, for a harmonic wave of the form, $\xi = \xi_0 \exp[i(\omega t - kx)]$

→ differentiate and substitute for the impedance z;

Sound waves in a gas

Derivation is similar to waves in a solid bar. Waves are again longitudinal.

- Same diagram:
- shows portion of the gas having cross section A



- This time the displacement ξ at x_0 causes a change in pressure dP

The measure of a gas's resistance to being compressed is called its;

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Sound waves in a gas (2)

The changes of pressure in the gas are too fast for the system to exchange energy with its surroundings, so the changes are adiabatic:

γ is the ratio…………
…………………………...

differentiate wrt V:

substitute def'n of B:

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Sound waves in a gas (3)

- As with the solid bar, we calculate the resultant force on the element, which this time we get from the difference in pressure:

$$P_{x_0} - P_{x_0+dx} = \left[P_{x_0} - \left(P_{x_0} + \frac{\partial P}{\partial x} dx \right) \right]$$

- Again use Newton's 2nd law for the element of gas :

remember: pressure = force acting on unit area

force per unit area

mass per unit area

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Sound waves in a gas (4)

$$-\frac{\partial P}{\partial x} dx = \rho dx \frac{\partial^2 \xi}{\partial t^2}$$

To get to a wave equation we need to find a relation between *P* and ξ. We do this through the definition of the bulk modulus.

First we note that; *V* = *A.dx* and *dV* = *A.d*ξ $\Rightarrow dP = -B\frac{d\xi}{dx}$

- Next, write the total pressure as being ambient pressure + change in pressure due to the wave:

$$P = P_0 + dP$$

- But since only *dP* changes as the wave passes through the medium, then

Sound waves in a gas (5)

$$\frac{\partial P}{\partial x} = -B\frac{d^2\xi}{dx^2}$$

$$-\frac{\partial P}{\partial x}dx = \rho dx\frac{\partial^2\xi}{\partial t^2}$$

- By substitution into our previous expression, and simplifying: $\frac{\partial P}{\partial x} = -B\frac{d^2\xi}{dx^2}$ $\quad -\frac{\partial P}{\partial x}dx = \rho dx\frac{\partial^2\xi}{\partial t^2}$

$\Rightarrow$

- which yields the dispersionless wave equation with phase velocity;

Sound waves in a gas (6)

The pressure through the gas acts as a type of wave:

- Using the harmonic solution to the wave equation for ξ: $\xi = \xi_0 \exp[i(\omega t - kx)]$
- So, remembering that; $dP = -B\frac{d\xi}{dx}$

then;

i.e. → the excess pressure *dP* is itself a wave, which ………………………………………………………………..

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Acoustic impedance

Again, to calculate impedance (z);

$$z = \frac{\text{drivingforce}}{\text{appropriate particle / medium velocity}}$$

By substitution; $z = \frac{dP}{d\xi / dt}$

So, for a harmonic wave of the form, $\xi = \xi_0 \exp[i(\omega t - kx)]$

→ differentiate and substitute to get impedance *z*;