

Lecture 16.

Lecture objectives.

- To be familiar with partial standing waves; how they form (e.g. from imperfect reflections $R < 1$) and their mathematical representation.
- To understand how wavepackets can form; i.e. due to superpositions of waves of different ω and k .
- To appreciate that the packet (envelope) can travel at a different velocity to the wavelets (i.e. group velocity is not necessarily the same as the phase velocity).
- To know (and remember) the equations for group and phase velocities.

Post-lecture tasks.

- Define the terms phase velocity, group velocity and dispersion of waves.
- A wave on a guitar string obeys the dispersion relation: $\omega = ak^2 + bk^3$. (a and b are constants). Write down expressions for the phase and group velocities of the wave.
- The dispersion relation for transverse waves on a 1D periodic structure is given by;

$$\omega = 2 \left(\frac{T}{ma} \right)^{\frac{1}{2}} \sin \left(\frac{ka}{2} \right)$$

Derive expressions for the phase and group velocities.

- In the dispersion relation $\omega = ck + dk^3$ for a wave on a piano string, where c and d are positive constants, calculate which is the larger value, the phase velocity or the group velocity.