# Differentiating exponentials

The exponential function $e^x$ is perhaps the easiest function to differentiate: it is the only function whose derivative is the same as the function itself.

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(e^u) = e^x \frac{du}{dx}$$

Strictly speaking, this is the correct general case

Slightly more complicated is $e^{2x}$ or $e^{3x}$ and so on. In these cases the number in front of the x "comes down" to the front in the derivative, (this is because we actually "bring down" the differential of this exponent)

$$\frac{d}{dx}(e^{2x}) = 2e^{2x} \quad \text{and} \quad \frac{d}{dx}(e^{3x}) = 3e^{3x}$$

For example, if the "exponent" is now a slightly more complicated function of x, see how we "bring down" the differential of this function; e.g. for exp $(x^2)$ etc.

$$\frac{d}{dx}(e^{x^2}) = 2x.e^{x^2} \quad \text{and} \quad \frac{d}{dx}(e^{x^3}) = 3x^2.e^{x^3}$$

If the function is in terms of time (t) instead of x, (as with some of the functions in our lecture course, e.g. $x = \exp(\omega t)$ ), then the differential works in the same way;

$$\frac{d}{dt}(e^{\omega t}) = \omega e^{\omega t}$$

Or for the complex displacement of our SHM system; i.e. $x = A.\exp j(\omega t + \phi)$

$$\dot{x} = \frac{d}{dt}(x) = \frac{d}{dt}(Ae^{j(\omega t+\phi)}) = A\underline{j\omega}e^{j(\omega t+\phi)}$$

Differential of raised exponent

# Practice Problems:

Complete the following:

$$\frac{d}{dx}(e^{2x^2}) =$$

$$\frac{d}{dx}(e^{x^2+2x}) =$$

$$\frac{d}{dt}(Ae^{j(\omega t+\phi)}) =$$

$$\frac{d}{dt}(Aj\omega e^{j(\omega t+\phi)}) =$$

$$\frac{d}{dt}(-je^{j(\omega^2 t+\phi)}) =$$

$$\frac{d}{dt}\left(\frac{Aje^{-j(\omega t+\phi)}}{\omega}\right) =$$