PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	**PHY2002**
Name of module	**Quantum Physics 1**
Date of examination	**January 2008**

1. Radioactive decay, scanning microscopes..

   Check general properties of barriers and probabilities distributions in course text.

   Approx expression is $P_{trans} \approx G\exp(-2|q|L)$ with $G = 16\frac{E}{V_0}\left(1-\frac{E}{V_0}\right)$ and $|q| = \sqrt{\frac{2m|E-V_0|}{\hbar^2}}$ provided $|q|L << 1$. Should find P_{trans} ~ $3 \times 10\text{-}^{10}$.

2. For operators that commute order of measurement of associated properties does not matter.

   Use standard formula for zero point energy, $E = \frac{1}{2}\hbar\omega_0$.

   3 nm.

3. Probabilities are P(1) = 3/7, P(2) = 1/7, P(3) = 0, P(4) = 1/7, P(5) = 2/7

   Expectation value is $\frac{12}{7}\hbar$

4. Energy approx 10^{-20} J, wavelength approx 10 μm, Q ~ 20.

5. Should find $\Psi_1 = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi x}{L}\right)$

   Probabilities are 0.82 and 0.5, found by appropriate integration.

UNIVERSITY OF EXETER – SCHOOL OF PHYSICS
SOLUTION TO EXAMINATION QUESTION

Module number	PHY2006
Year of Examination	2007–08
Question Number	
Name of Setter	
Initial of Checker	

HINTS & SOLUTIONS

Q1 (i) Book work
(ii) Book work

Q2 (i) Book work
(ii) $\oint_S \vec{E}\cdot d\vec{S} = \frac{1}{\epsilon_0}\int \rho\, dV \Rightarrow 2\pi r l E = \frac{1}{\epsilon_0}\int_0^{2\pi} d\phi \int_0^{l} dz \int_0^{r} (ks)\, s\, ds$

$\therefore \vec{E} = \frac{k}{3\epsilon_0} r^2 \hat{r}$

Numerical value $E = 0.0188$ N/c

Q3 All book work

Q4 (i) Book work
(ii) Book work
(iii) Book/home work
Numerical solution $|A_z| = \frac{\mu_0 I}{2\pi} \ln\left(\frac{4}{2}\right) = 13.8629 \times 10^{-7}$ T.m

Q5 All book work

1. PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

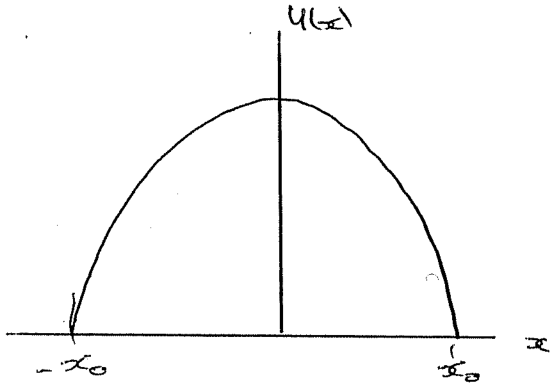
Module Code	**PHY2007**
Name of module	**Relativity II and Mechanics**
Date of examination	**June 2008**

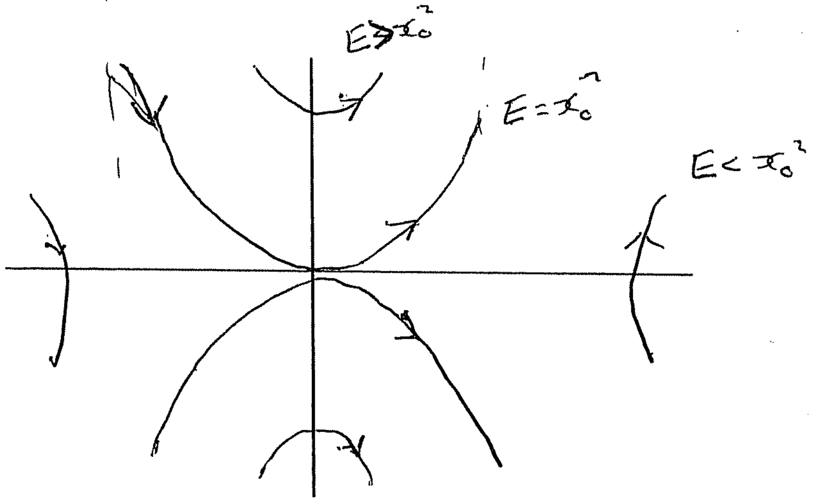
1. First parts bookwork.
   Torque = 11.10^5 Nm. (substitution – make sure units are correct including angular speed in rad s^{-1})
   Equate work (power x time) to kinetic energy => t = 2205 s.

2. First parts book work. Use parallel axes theorem to show I about corner is $7\rho a^4/12$.
   Kinetic energy = $7\rho a^4\dot{\theta}^2/12$. Potential energy $\approx \rho g a^3\theta/2\sqrt{2}$. Set time derivative of total energy to zero to recover eqn for SHM.

3. First parts bookwork. For projectile $L = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2 - mgy$ => $\ddot{x} = 0$, $\ddot{y} = -g$. Integration and substitution => $y = -\frac{1}{2}g(x/C)^2 + Ax/C$where A and C are constants of integration.

4. Trajectories:





5. First parts bookwork. Energy = 305 Mev, angle = $\cos^{-1}$ 0.95.

For final part, conservation of energy and momentum => $E_p^2 = p_p^2 c^2\cos^2\theta$, where E_p and p_p are the total energy and momentum of the particle, but we also require $E_p^2 > p_p^2c^2$, which is impossible.

PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

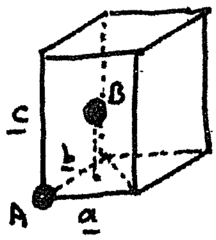
Module Code	PHY2009
Name of module	Physics of Crystals
Date of examination	January 2008

1. (i) Description of bonds & resulting physical properties – see lecture notes.

Examples : metallic bonding – Copper
van der Waals " – Solid Argon

(ii) hcp structure :
- take 2D triangular structure (as formed by packing billiard balls in a plane)
- stack successive sheets in ABABAB sequence, as distinct from ABCABC sequence which forms fcc structure

Consider hcp unit cell, which contains 2 basis atoms at A, B.



$\underline{r}_A = 0 \;, \; \underline{r}_B = \frac{1}{3}\underline{a} + \frac{1}{3}\underline{b} + \frac{1}{2}\underline{c}$

Spheres of maximum radius touch half way along AB

$\therefore \; \frac{1}{4} \underline{r}_B^2 = \underline{a}^2 = \underline{b}^2$

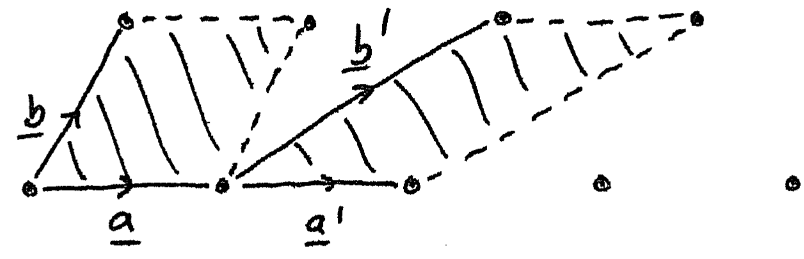
$\therefore \; |\underline{c}| = 2\sqrt{\frac{2}{3}}\,|\underline{a}|$

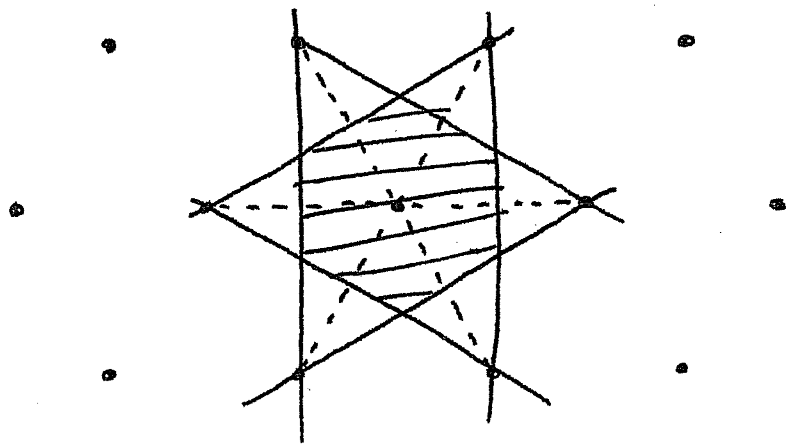
$$\text{Packing fraction} = \frac{2 \times \frac{4}{3}\pi\left(\frac{a}{2}\right)^3}{(\underline{a} \times \underline{b}) \cdot \underline{c}} = \underline{0.740}$$

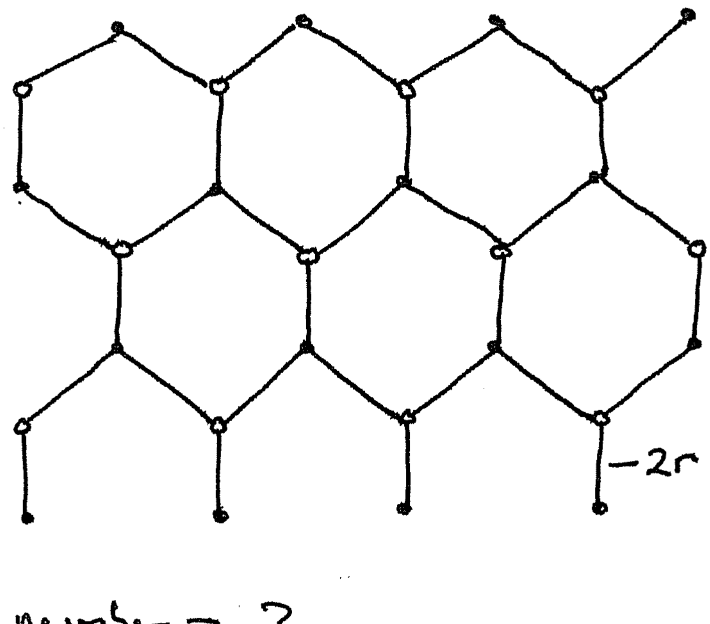
PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	PHY2009
Name of module	Physics of Crystals
Date of examination	January 2008

2.







Coordination number = 3

$$\text{Packing fraction} = \frac{2 \times \pi r^2}{|\underline{a} \times \underline{b}|} = \frac{\pi}{3\sqrt{3}}$$

PHYSICS EXAMINATION PROBLEMS
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Module Code	PHY2009
Name of module	Physics of Crystals
Date of examination	January 2008

3. Cu_3Au

Simple cubic

$(0,0,0)$ Au, $(\frac{1}{2},\frac{1}{2},0), (\frac{1}{2},0,\frac{1}{2}), (0,\frac{1}{2},\frac{1}{2})$ Cu

$\theta = \sin^{-1}\left(\frac{\lambda}{2d}\right)$ where $d = \frac{a}{\sqrt{h^2+k^2+l^2}}$ and h, k, L can take any integer values

$\therefore$ if $\lambda = 0.154$ nm, $\theta =$ 11.9°, 16.9°, 20.9°, 24.3° ....

Structure is fcc after heating $\therefore$ consider fcc structure factor

Only the 3rd & 4th peaks now observed.

PHYSICS EXAMINATION PROBLEMS
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Module Code	PHY2009
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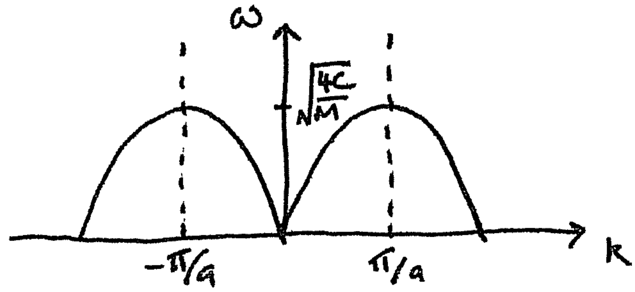
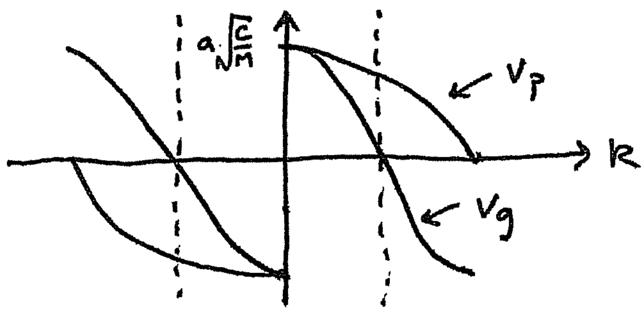
4. Derivation of equations of motion and dispersion relation is in the lecture notes



$$V_p = \frac{\omega}{k}, \quad V_g = \frac{\partial \omega}{\partial k} = a\left(\frac{C}{M}\right)^{1/2} \cos\left(\frac{ka}{2}\right) \text{ for } 0 < k < \frac{2\pi}{a}$$



Speed of sound: $k \simeq 0$, $V_p = V_g = a\sqrt{\frac{C}{M}} = \underline{9360 \text{ ms}^{-1}}$

PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	PHY2009
Name of module	Physics of Crystals
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5. $N(k) = \frac{k^3}{3\pi^2}$ and $E = \frac{\hbar^2 k^2}{2m}$ $\therefore N(E) = \frac{1}{3\pi^2}\left(\frac{2mE}{\hbar^2}\right)^{3/2}$

Sodium: 2 electrons contained within volume $(0.423\text{ nm})^3$

$\therefore$ can cachulate $N(E_F) = \frac{2}{(0.423\text{ nm})^3}$

Hence $E_F = 3.22$ eV

$V_F \simeq (2E_F/m)^{1/2} = 1.1 \times 10^6$ m/s

Repeated, reduced and extended zone schemes described in lectures notes as were Fermi level positions of different classes of electrical materials.

PHY2013

ANATOMY AND PHYSIOLOGY

FOR DISCUSSION WITH PERSONAL TUTORS, THEREFORE NO HINTS & TIPS ARE PROVIDED.

THE MODULE LEADER WILL PROVIDE REVISION PROBLEMS AND TUTORIAL AT THE END OF THE COURSE.

Hints and Tips for PHY2018, Mathematics with Physical Applications

1. (i) Complementary solution is $y = a + b\exp(2x) + c\exp(-2x)$. Try particular solution $d\exp(-x)$ from which $d = 2$. Solution is $y = a + b\exp(2x) + c\exp(-2x) + 2\exp(-x)$. To satisfy boundary conditions, require$b = 1/4, c = -3/2$ and $a = -3/2$.

   (ii) For $m = 1$, $n = n_0 \exp(-kt)$ and for $m = 2$, $n = \frac{1}{c+kt} = \frac{n_0}{1+kn_0t}$

   Plot $\ln(n)$ versus t in first case and $1/n$ versus t

2. $$\frac{1}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) = l(l+1)$$

   Try the solution $R = ar^m$. Then $m(m+1) = l(l+1)$ and hence $m = l$ or $m = -l-1$. Thus general solution is

   $$R = Ar^l + \frac{B}{r^{l+1}}$$

   Wronskian is the determinant of the matrix

   $$\begin{pmatrix} r^l & r^{-l-1} \\ lr^{l-1} & (-l-1)r^{-l-2} \end{pmatrix}$$

   or $-(l+1)\frac{1}{r^2} - \frac{l}{r^2} = -\frac{(2l+1)}{r^2}$. As Wronskian is not zero, functions are independent. As potential is finite at origin and as $\cos^2\theta = 2/3P_2 + 1/3P_0$, solution of Laplace's equation which is $V_0\cos^2\theta$ on surface $r = a$ is

   $$\frac{2V_0}{3}P_2(\cos\theta)(r/a)^2 + \frac{V_0}{3}P_0(\cos\theta)$$

   for $r \leq a$ and

   $$\frac{2V_0}{3}P_2(\cos\theta)(a/r)^3 + \frac{V_0}{3}P_0(\cos\theta),$$

   for $r \geq 0$.

3. Heat equation

   $$\kappa\nabla^2 u = \frac{\partial u}{\partial t}$$

   Set $u = 100 - 100x/l + v(x,t)$ and note that as $u(x,0)$ is $100x/l$, then for $0 < x < l$, $v(x,0) = -100 + 200x/l = -200(l-x)/l + 100$, while $v(0,t) = v(l,t) = 0$..

Qustion 4

a) $A\underline{X} = \underline{Y}$ $\quad A = \begin{pmatrix} -1 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 3 \end{pmatrix} \quad \underline{X} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \quad \underline{Y} = \begin{pmatrix} 0 \\ 0 \\ 8 \end{pmatrix}$ [2]

b) det(A)=-8 $\quad$ det(A) $\neq 0$ => there exists a unique solution [4]

c) Inverse matrix: $A^{-1} = \dfrac{(A_{jk})^T}{\det(A)}$ ← matrix of cofactors [5]

$$A^{-1} = \begin{pmatrix} 5 & 2 & -1 \\ 2 & -4 & -2 \\ -1 & -2 & -3 \end{pmatrix} \frac{1}{-8} \qquad AA^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

d) $\underline{X} = A^{-1}\underline{R} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ [5]

e) $\alpha = \arccos\left(\dfrac{X \cdot Y}{|X||Y|}\right) = 0.64$ rad [5]

f) det(A) [4]

```
det(A)
inv(A)
inv(A)*A
X=inv(A)*R
Alpha=acos(Y*X/(sqrt(X(1)^2+X(2)^2+X(3)^2)* sqrt(Y(1)^2+Y(2)^2+Y(3)^2)))
```

Question 5.

Det $(A-I\lambda)=0$ → characteristic equation $(8-\lambda)(2-\lambda)=0$ → $\lambda_{1,2}= 2,8$. [4]

Eigenvectors:

$\lambda_1=2$: $\begin{cases} 3x_1 + 3x_2 = 0 \\ 3x_1 - 3x_2 = 0 \end{cases}$ let $x_1=1$ then $x_2=-1$

$\lambda_2=8$: $\begin{cases} -3x_1 + 3x_2 = 0 \\ 3x_1 - 3x_2 = 0 \end{cases}$ let $x_2=1$ then $x_1=1$

Eigenvector matrix: B= $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ principle directions [5]

Orthogonal to each other: e.g. $(1,1)*\begin{pmatrix}1\\-1\end{pmatrix}=0$ [2]

Ellipsoid. With ratio of axes: 8/2 [2,4]

Diagonal form of A: $\begin{pmatrix}\lambda_1 & 0 & 0\\ 0 & \lambda_2 & 0\\ 0 & 0 & \lambda_3\end{pmatrix}=B^{-1}AB$ [2,2]

In MATLAB to find eigenvalues and eigenvectors of A:

[B,L]=eig(A) [2]
where B matrix of unit eigenvectors and L diagonal matrix of eigenvalues

Question 6.

a) Given the boundary conditions basis functions should be of the form $k_n=\frac{2\pi n}{L}$ [5]

b) Normalisation: $\int_0^L f_n(x)f_n(x)dx=1$

$$A^2\int_0^L dx=A^2L=1$$

→ $A=\frac{1}{\sqrt{L}}$ [5]

Orthogonality ($n\neq m$): $\int_0^L f_n(x)f_m(x)dx=0$

$$\frac{2}{\pi}\int_0^L \exp(i(k_m-k_n)x)dx=0$$

c) Given: $\hat{H}(x)\equiv-\frac{\hbar^2}{2M}\frac{d^2}{dx^2}$, $\Psi(x)=\sum_{n=1}^{\infty}c_nf_n(x)$ [2]

the differential equation can be rewritten in the following form:

$$\begin{pmatrix}H_{11} & H_{12} & . & .\\ H_{21} & H_{22} & . & .\\ . & H_{nm} & . & .\\ . & . & . & .\end{pmatrix}\begin{pmatrix}c_1\\c_2\\c_3\\.\end{pmatrix}=E\begin{pmatrix}c_1\\c_2\\c_3\\.\end{pmatrix}$$ → Eigenvalue problem [3]

d)Where matrix elements are found as

$H_{nm}=\int_0^L f^*{}_n(x)(-\frac{\hbar^2}{2M}\frac{d^2}{dx^2}f_m(x))dx=\frac{\hbar^2}{2M}k_n^2\delta_{m,n}$ [5]

The matrix is diagonal so the system is in the canonical form. The wavefunctions will be simply the basis set components: i.e.

$$\psi_1(x) = \frac{1}{\sqrt{L}}\exp(ik_1 x)$$

[5]

$$\psi_2(x) = \frac{1}{\sqrt{L}}\exp(ik_2 x)$$

PHY2019

OBSERVING THE UNIVERSE

NO HINTS & TIPS ARE PROVIDED.

FOR HELP, PLEASE CONTACT THE MODULE LEAD.

PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	**PHY2020**
Name of module	**Lasers and Materials for Quantum Applications**
Date of examination	**June 2008**

3. $\Delta\nu/\delta\nu = 5$

4. $N_3/N_2 = 2.79 \times 10^{-41}$

PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	**PHY2201**
Name of module	**Statistical Physics**
Date of examination	**Jan 2008**

1. i) See course notes. $\Delta S_{total} = -\frac{Q_1}{T_1} + \frac{Q_2}{T_2} \Rightarrow$ if $Q_2 = 0 \Rightarrow \Delta S < 0$, contrary to the 2nd Law, unless $T = 0$.

   ii) $P_{\min} = Q_2 \frac{T_1 - T_2}{T_2} \approx 0.3$ MW.

   iii) Use $dS_1 = \frac{m c_V dT}{T}$ for water and $\Delta S_2 = -m c_V \frac{\Delta T}{T_f}$ for the reservoir, where $T_f = T_{reservoir}$

   $\Delta S_{total} = m c_V \left[\ln\left(\frac{T_f}{T_i}\right) + \frac{T_i}{T_f} - 1 \right] \approx 122\ \mathrm{JK^{-1}}$.

2. i) See course notes. Equilibrium: $\langle v_x \rangle = \langle v_y \rangle = \langle v_z \rangle = 0 \Rightarrow$ no bulk motion..

   ii) See course notes.

   $\langle \varepsilon \rangle = \int_0^\infty \varepsilon\, p(\varepsilon)\, d\varepsilon = \frac{3}{2\beta} = \frac{3k_B T}{2} \Rightarrow \beta = \frac{1}{k_B T}$.

   Conduction electrons – 'quantum gas', rules governing occupancy dominate behaviour.

3. i) See course notes.

   ii) $T = \frac{\Delta \varepsilon}{k_B \ln 10} = 2.5 \times 10^4$ K.

   iii) $\Omega_{n,k} = \frac{(n+k-1)!}{n!k!} = 35$. Equally shared energy: $p = \frac{1}{35} \approx 0.029$; $S = 0$. Energy shared between two systems: $p = \frac{6}{35} \approx 0.17$.

4. i) See course notes.

   ii) See course notes.

   iii) Use $Z = \sum_{i=0}^{\infty} \exp\left(-\frac{\varepsilon_i}{k_B T}\right)$ and the geometrical series summation rule.

5. i) $S = k_B \ln \Omega$; see course notes. $\Omega_{A+B} = \Omega_A \cdot \Omega_B \Rightarrow S_{A+B} = k_B \ln \Omega_A + k_B \ln \Omega_B = S_A + S_B$

   a) $S = 0$: b) $S = k_B \ln N \approx 3.2 \times 10^{-23}\ \mathrm{JK^{-1}}$;

   c) $S = k_B \ln \Omega_{N/2} = k_B \ln \frac{N!}{(N/2)!(N/2)!} \approx N k_B \ln 2 \approx 7.63 \times 10^{-23}\ \mathrm{JK^{-1}}$.

   ii) See course notes.

   Classical limit: $\exp\left(\frac{E_i - E_F}{k_B T}\right) >> 1$. Therefore, $\frac{n_i}{w_i} \propto \exp\left(-\frac{E_i}{k_B T}\right)$.

PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	**PHY2208**
Name of module	**Optics**
Date of examination	**June 2008**

1. (iii) $\frac{1}{s}+\frac{1}{s'}=\frac{2}{R}$, but the image distance is infinite so $\frac{1}{s}+\frac{1}{\infty}=\frac{2}{0.2}$ and s = 0.1 m.

2. $D=d+\theta L=d+\frac{L\lambda}{d}$. Differentiate to give $\frac{dD}{dd}=1-\frac{L\lambda}{d^2}$ and set to zero for min so $d=\sqrt{\lambda L}$

3. $Z_R=25\text{cm}$

   $\theta=\frac{1.22\lambda}{d}=305\times10^{-9}\,radians$ giving distance of 10^5 Mpc.

4. Quarter wave plate so $(n_e-n_o)d=\frac{\lambda}{4}$ and $d=1.25\times10^{-4}\,m$

5. $2nd=m\lambda$ so m = 2500. For FP *R=mF*, so $10^8=2500F$ and $F=40000$