PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	**PHY1002**
Name of module	**Thermal Physics**
Date of examination	**June 2008**

1 $\Delta T = \frac{Q}{cm} = \frac{2300}{4186 \times 0.7} = 0.785 \text{ C}$

Therefore, the temperature is

$T = 37 - 0.785 = 36.2 \text{ C}$.

2 **(i) Venus atmosphere**

From $PV = NkT$ we find that $\frac{N}{V} = \frac{P}{kT}$. Therefore

$$\frac{(N/V)_{\text{Venus}}}{(N/V)_{\text{Earth}}} = \frac{(P/T)_{\text{Venus}}}{(P/T)_{\text{Earth}}} = \frac{9 \times 10^6/740}{1 \times 10^5/300} = 36.5$$

(ii) Gas in a cylinder

From the ideal gas equation of state $T = \frac{PV}{nR}$. But $P = \frac{F}{A} = \frac{k\Delta x}{A}$ and $V = A\Delta x$, where A is the piston area and k is the spring constant. Therefore

$$T = \frac{k\frac{\Delta x}{A}A\Delta x}{nR} = \frac{k(\Delta x)^2}{nR} = \frac{4.2 \times 10^4 \times (0.12)^2}{0.6 \times 8.314} = 121.2\,\text{K}$$

(iii) Isothermal compressibility of a van der Waals gas

Hint: first calculate $(\partial P/\partial V)_T$ *and then use the equation* $(\partial V/\partial P)_T = \frac{1}{(\partial P/\partial V)_T}$.

$$\kappa = \frac{V^2(V-b)^2}{RTV^3 - 2a(V-b)^2}$$

3 **(i) Radiating cylinders**

$N = 7$

(ii) Conduction through metal rods

(a) $$\left(\frac{L_1}{L_2}\right) = \sqrt{\frac{k_1}{k_2}\frac{\rho_2}{\rho_1}} = \sqrt{\frac{420}{14}\times\frac{7715}{10500}} = 4.69$$

(b) $$\left(\frac{R_1}{R_2}\right) = \sqrt{\frac{k_2}{k_1}\frac{L_1}{L_2}} = \sqrt{\frac{14}{420}\times 4.69} = 0.40$$

4 (b) $T = 102.6$ K

(c) $W = 2461.8$ J

(d) $W = -1372.9$ J

(e) For the adiabatic process $\Delta U = nC_V\Delta T = -2461.8\,\mathrm{J}$.

For the isothermal process $\Delta U = 0$.

5 $n = 7.46 \times 10^{-3}$ mol

PHYSICS EXAMINATION PROBLEMS
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Module Code	**PHY1003**
Name of module	**Properties of Matter**
Date of examination	**January 2008**

1

(c) $L = 2.72$ m

(d) The stresses in the two parts of the rod are the same and equal to 2.49×10^8 Pa.

(e) The strain for the copper rod is 2.26×10^{-3}.
The strain for the steel rod is 1.25×10^{-3}.

2

(c) $a = -16$ m s^{-2} (negative, as it is directed upwards)

(d) $t = 0.62$ s

3

(i) Mass of water supported my surface tension in a capillary

$m = 3.69 \times 10^{-6}$ kg $= 3.69$ mg

(ii) Capillary effects in a mercury barometer

The level is depressed by 3.29 mm, therefore this must be added to the reading.

(iii) Capillary formed between two concentric tubes

$$h = \frac{2\gamma\cos\theta}{\rho g(r_2 - r_1)}$$

4

(b) $$B = \frac{e^2 r_0^{11}}{48\pi\varepsilon_0}$$

(c) $$F(r) = -\frac{e^2}{4\pi\varepsilon_0 r^2} + \frac{12B}{r^{13}}$$

5

(ii) (a) $E_0 = 4.39 \times 10^{-20}$ J
$E_1 = 1.32 \times 10^{-19}$ J
$E_0 = 2.20 \times 10^{-19}$ J

$\lambda_{1\to0} = 2.26 \times 10^{-6}$ m $= 2.26$ μm

$\lambda_{2\to1} = \lambda_{1\to0}$ as the energy gap between adjacent states is the same.

PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	PHY1104
Name of module	Fundamental Electromagnetism I
Date of examination	June 2008

1. (i) $\vec{F} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}\hat{r}$ definitions in text

(ii) (a) $\vec{F} = 0\hat{x} + 0\hat{y}$

(b) $\vec{F} = \frac{1}{4\pi\varepsilon_o}\frac{qQ}{R^2}\left[\frac{\sqrt{3}}{2}\hat{x} + \frac{1}{2}\hat{y}\right]$

(c) All remaining 5 charges act in same direction as force in part b

(d) $\vec{E} = \left[\frac{\sigma}{\varepsilon_o} + \frac{Q}{4\pi\varepsilon_o R^2}\right]\hat{r}$

(e) Gauss' Law $\int \vec{E}.d\vec{A} = \frac{Q_{enc}}{\varepsilon_o}$ definitions in text

2. (i) $\sum i = 0$ (charge), $\sum V = 0$ (energy)

(ii) (a) $I_1 = 5A,\ I_2 = -3A,\ I_3 = 2A$

(b) middle branch battery charging and could represent a real battery

(c) 14V

(iii) (a) 6.5p

(b) $9.5\text{x}10^4$ C

3. (i) $d\vec{B} = \frac{\mu_o}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}$ definitions in text

(ii) (a) no contribution from horizontal wire

(b) $\int dB = \frac{\mu_o I}{4\pi}\int \frac{d}{\left(y^2 + d^2\right)^{3/2}}\,dy$

(c) vertical wires F=IyB in plane of paper pointing outside loop
horizontal wires F=IxB in plane of paper pointing outside loop

(d) $\vec{\tau} = \vec{r}\times\vec{F}$ No torque on loop, forces cancel

4. (i) $\oint \vec{B}.d\vec{l} = \mu_o I_{enc}$ definitions in text

(ii) (a) $B = \frac{\mu_o I r}{2\pi r_1^2}$ field lines clockwise

(b) $B = \frac{\mu_o I}{2\pi r}$ field lines clockwise

(c) $B = -\frac{\mu_o I}{2\pi r}$ field lines counterclockwise

(iii) (a) $\Phi_B = \int \vec{B}.d\vec{A}$ magnetic flux, definitions in text

$\varepsilon = -\frac{d\Phi_B}{dt}$ Faraday-Lenz Law, sign convention explained in text

(b) Explained in text

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5. (i) (a) Use Kirchoff's Law in a circuit loop to get starting point: $\varepsilon = \frac{Q}{C} + IR$

Substitute definition of current and integrate resulting expression

(b) Use I=dQ/dt to get I, then solve for Power and Work

(c) RC, Consider charge after time RC

(ii) Draw loop including inside edge of capacitor and just outside and apply Faraday's Law

PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	**PHY1105**
Name of module	**Relativity and Vectors 1**
Date of examination	**January 2008**

1. Use of Newton's second law, conservation of momentum and conservation of Energy.

(i) For apparent weight work out net acceleration and use this in Newton's second law.
(ii) Use conservation of momentum in the two initial directions of motion.
(iii) Use change in KE = KE after – KE before (fraction lost is 62.5%)

2. (i) Circular motion question.

(ii) Effectiveness of centrifuge depends on acceleration $\propto \frac{\omega^2}{r}$, Centratis is best

(iii) Angle does not depend on mass.

3. SR question

(iii) u/c is 0.66

4. Forces and Potentials.

(i) a) answer is 5.6 cm. b) answer is 1.1J.
(ii) a) 8 cm, b) 0.5 mm, c) 1.6×10^{-4} J

5. SR question.

(ii) Use total energy of the two protons before collision = rest energy of two protons + rest energy of pion, i.e. $\frac{2m_pc^2}{\sqrt{1-v^2/c^2}} = 2m_pc^2 + m_\pi c^2$

PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code and Lecturer	PHY1106: PV
Name of module	Waves and Oscillations
Date of examination	June 2008

1. Standard expressions for impedances of R, L, and C (from lecture notes). Explain that voltage is divided in proportion to impedances of components etc. (from notes).

Circuit resonates at Z_min; i.e. at resonance, $\omega L - \frac{1}{\omega C} = 0$ and rearrange to $\omega_0 = \frac{1}{\sqrt{LC}}$.

Standard lecture notes for meaning of ϕ and equation for cos ϕ.

Use the formula for electrical power; $P_{av} = \frac{V_0^2}{2|Z|}\cos\phi$ to calculate dissipated powers:

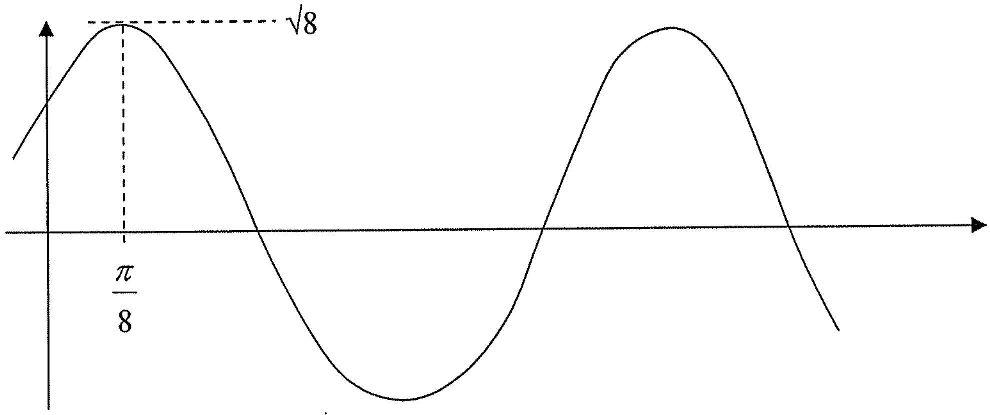
i.e. for L = 0.2 H, P_{av} = 26.3 W
for L = 2 H, P_{av} = 40 W
and for L = 20 H, P_{av} = 0.8 W

Resonance occurs for L = 2 H and so max power is transferred.

2. ω = 2 /s and T = π s.

Standard trig bookwork to isolate and calculate constants A and B. A = -2 and B = -2

Similar trig bookwork to do same for this expression and calculate C and ϕ.
C = root 8, $\phi = \pi/4$

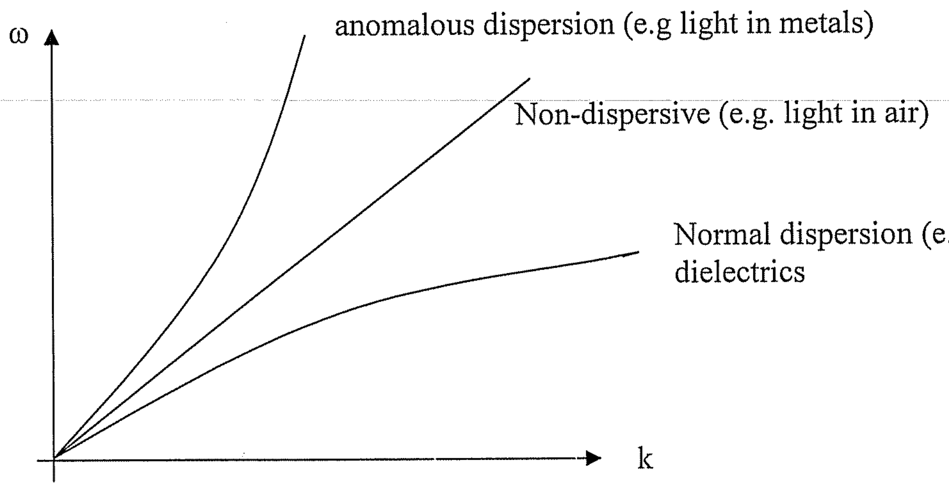


For total energy, use $E = \frac{1}{2}kA^2 = 40$ J.

3. $\lambda = \frac{2\pi}{k}$ $v_{ph} = \frac{\omega}{k}$ $v_g = \frac{d\omega}{dk}$

A medium is non dispersive when $v_g = v_{ph}$

Anomalous dispersion when $v_g > v_{ph}$ Normal dispersion when $v_g < v_{ph}$



Relation for group velocity: $v_g = \frac{d\omega}{dk}$

Use standard relation for phase velocity: $v_{ph} = \frac{\omega}{k} = c\frac{\sin{}^{ka}\!/_{2}}{{}^{ka}\!/_{2}}$.

Standard differentiation required to verify expression for group velocity.

At long wavelengths, $\lambda \to \infty$ and therefore $k \to 0$.
Also, as $x \to 0$, $\sin x \to x$ and $\cos x \to 1$.
Use these to predict that V_ph and V_g tend to the limiting value of c.

4. amplitude = 0.01 m; ang. freq. = 20 /s; wavenumber = 10 /m.

wavelength = 0.63 m v_ph = 2 m/s

max transverse velocity = amp $\times$ ang. freq. = 0.2 m/s

Use $v = \sqrt{\frac{T}{\rho}}$ to calculate tension; T = 1.2 N.

Impedance of tube = $Z = \sqrt{\rho T}$ = 0.6 kg /s

Relation for group velocity: $v_g = \frac{d\omega}{dk}$

Use standard relation for phase velocity: $v_{ph} = \frac{\omega}{k} = \left(\frac{T}{\rho} + 4k^2\right)^{1/2}$.

Standard differentiation required to verify expression for group velocity.

Calculate group velocity to be v_g = 40 m/s.

5. $\omega_1 = 3.200 \times 10^{15}$ /s $\omega 2 = 3.197 \times 10^{15}$ /s

$k_1 = 9.31$ N/m $k_2 = 9.33$ N/m

Estimate of wavelength interval for D2 line (associated with bandwidth), $\Delta\lambda = 0.3$ nm

Use standard calculus manipulation to derive formula, then apply to calculate $\Delta\omega_{D2} = 1.6 \times 10^{12}$ /s

Hints and Tips for PHY1115, Mathematatcal Skills

1.

$$z = 4 + 5i = 6.403e^{i0.89605}, \; z^2 = -9 + 40i, \; zz^* = 41,$$
$$1/z = \frac{4-5i}{41} = 0.0976 - 0.1220i, \; z^{1/2} = \pm(2.2806 + 1.0962i),$$
$$\ln z = 1.8568 + i0.8961, \; exp(z) = 15.4874 - 52.355i$$

2. Put $y = \exp(2x)$

$$3y^2 - 5 = 12y, \;\; 3y^2 - 12y - 5 = 0$$
$$y = 4.380 \; or \; -0.380,$$
$$\exp(2x) = 4.380.$$

Hence $x = 0.738$.

3.

$$\frac{dt}{dx} = 1/x, \;\; \frac{dy}{dt} = 12t^2 + 2$$
$$\frac{dy}{dx} = (12t^2 + 2)e^{-t}$$

$$\frac{d\tan(x)}{dx} = \sec^2(x)$$
$$\frac{d^2\tan(x)}{dx^2} = 2\sec^2(x)\tan(x)$$
$$y = 1 + 2(x - \pi/4) + 2(x - \pi/4)^2$$

$\tan(0.98540) = 1.50850$ and polynomial gives 1.4800051

4. $|a|$, $|b|$, $a.b$, and θ are 6.70820379, 5.38516474, 14., 67.1981888°. while $\cos\theta$ is 0.38754545.

   $\mathbf{a} \wedge \mathbf{b} = (22, 24, -7)$.

5.

$$A = \tan^{-1}(x) - \tanh^{-1}(x)$$
$$\frac{dA}{dx} = \frac{1}{1+x^2} + \frac{1}{x^2-1} = \frac{2x^2}{x^4-1}$$

Turning point at $x = 0$ and as A is odd function, then $\frac{d^2A}{dx^2} = 0$ at $x = 0$. Hence point of inflection.

6.

$$\int fg dx = fG - \int Gf' dx$$

Integral is $(-t^2 - 2t - 2)\exp(-t)$.

7.

$$z = 2i \pm \frac{1}{2}\sqrt{-16 - 16} = 2i \pm 2i\sqrt{2}$$

Substituting in gives

$$z_1^2 - 4i\, z_1 + 4 = 0$$
$$z_2^2 - 4i\, z_2 + 4 = 0$$

8. Curve looks like inverted tumbler with maximum at origin.

$$\frac{dy}{dx} = \frac{2xe^{x^2}}{(4 + e^{x^2})^2}$$

This vanishes at $x = 0$ and is y is clearly a minimum. The minumum value is 0.2.

9. (a) $20 \times 19 \times 18 \times 17 = 116280$ (b) $116280/4! = 4845$

10.

$$x = e^{pt}$$
$$5p^2 + 29p - 6 = 0$$
$$p = 1/5, \quad -6$$
$$x = a\exp(t/5) + b\exp(-6t)$$

Particular solution

$$x = A\cos(t) + B\sin(t)$$

$$5\frac{d^2x}{dt^2} + 29\frac{dx}{dt} - 6x = (-5A + 29B - 6A)\cos(t) + (-5B - 29A - 6B)\sin(t) = \sin(t)$$
$$A = (29/11)B, \quad B = -1/(29^2/11 + 11) = -11/962, \quad A = -29/962$$

Soln

$$x = a\exp(t/5) + b\exp(-6t) - \frac{29}{962}\cos(t) - \frac{11}{962}\sin(t)$$

PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	**PHY1116**
Name of module	**MATHEMATICS FOR PHYSICISTS**
Date of examination	**June 2008**

1. (i) $\sin x - x\cos x \approx \dfrac{x^3}{3}$ and $x\sin x \approx x^2$; limit is 0 (ii) hint is in question

2. (i) $\mathbf{A}^{-1} = \begin{pmatrix} -2 & 1 \\ 3/2 & -1/2 \end{pmatrix}$ (ii) set determinant to zero. $\alpha = 1$

3. (a) $f_\varphi = \rho\cos\varphi; f_\rho = \sin\varphi; f_{\varphi\varphi} = -\rho\sin\varphi; f_{\rho\rho} = 0; f_{\rho\varphi} = f_{\varphi\rho} = \cos\varphi$

   (b) $f(x,y) = y; f_x = 0; f_y = 1$.

   (c) No hint needed

4. Use circular polars: $M = 2\sqrt{2}m_0a^2$

5. Use spherical polars: $M = \dfrac{2\pi m_0 a^6}{3}$

6. (i) Volume $= \mathbf{a}\bullet\mathbf{b}\times\mathbf{c} = 4$; (ii) –0.5

7. (i) No hint required (spherical cords slightly easier) (ii) Integral = 16π

8. $y = \dfrac{2}{3} + \dfrac{C}{x^3}; y = \dfrac{2}{3}\left(1 - \dfrac{1}{x^3}\right)$

9. Use the fact that the function is odd to avoid one integral; put x = π/2 in last part.

10. (a) Odd.

    (b) Remember that integrals involving delta-functions don't need to be integrated. FT is imaginary because function is odd.

PHY1118

UNIVERSITY OF EXETER

SCHOOL OF PHYSICS

QUANTUM AND ASTROPHYSICAL PHENOMENA

January 2008

Hints and tips

1. vii) Convert 500 nm to energy in eV; subtract 0.6 eV. What energy remains?

2. ii) Differentiate the expression relating λ, f and c.

   iii) Convert $\Delta\lambda$ to Δf, then use uncertainty relation involving Δt.

   iv) Do not forget that this is a three dimensional problem.

3.