UNIVERSITY OF EXETER - SCHOOL OF PHYSICS
SOLUTION TO EXAMINATION QUESTION

Module Number	PHYM 401
Year of Examination	2007
Question Number	3-4
Name of Setter	AKS
Initials of Checker	HINTS AND SOLUTIONS

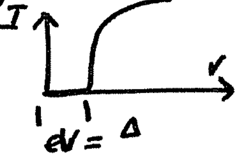
(3.) → Defs in the course.

→ $B_c(T) = B_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]$ → $B_c(0.5K) = 8.18\,mT$

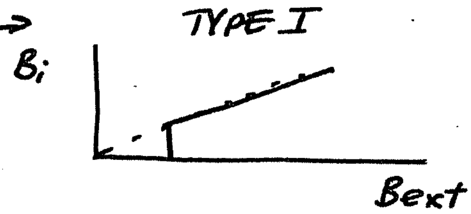
→ Unusual properties are in the course

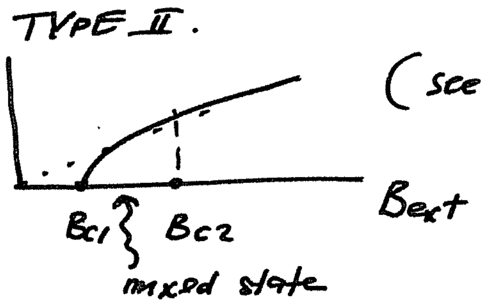
→ Δ decreases with increasing T to become 0 at T_c.

→ The gap is measured by tunnelling in S-I-N junction:



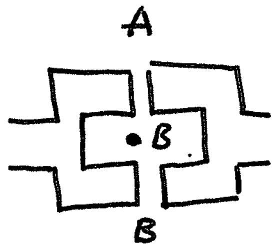
→





(see the course)

→



→ Josephson's result: $J = J_0\left(\sin\left(\delta_0 + \frac{e\Phi}{\hbar}\right) + \sin\left(\delta_0 - \frac{e\Phi}{\hbar}\right)\right)$

$\propto \cos\frac{e\Phi}{\hbar} = \cos\frac{\pi\Phi}{\Phi_0}$

→ $\Delta\Phi = 2\Phi_0 = A\,\Delta B$

If A is increased by 4 times, the period will <u>decrease</u> by 4 times.

(4.) (i) → Defs in the course. → $g(\varepsilon) = a\,\varepsilon^{1/2}$, $n \propto g^3(E_F) \rightarrow g(E_F) \propto n^{1/3}$

$k_s^2 = b\,g(E_F) \rightarrow \lambda = \frac{1}{k_s} \propto n^{-1/6}$

(ii) → Defs in the course.

→ $L = rmv$, $M = IA = \frac{e}{2m}L$; Thus $\gamma = \frac{M}{L} = \frac{e}{2m} = \mu_B/\hbar$

while for spin: $M = \mu_B \rightarrow \gamma = \frac{e}{m}$

→ For conducting electrons:

$$\chi \sim \frac{\mu_0\mu_B^2 n}{k_B T} \times \left(\frac{k_B T}{E_F}\right)$$

↖ fraction of magnetised electrons

→ paramagnetism in metals is <u>small</u>, and <u>T-independent</u>.

UNIVERSITY OF EXETER - SCHOOL OF PHYSICS
SOLUTION TO EXAMINATION QUESTION

Module Number	PHYM401
Year of Examination	2007
Question Number	1-2
Name of Setter	AKS
Initials of Checker	HINTS AND SOLUTIONS

1. (i) → Definitions in the course; the width of the band is 4γ.

→ $m^{*-1} = \frac{1}{\hbar^2}\frac{d^2E}{dk^2} = \frac{2\gamma a^2}{\hbar^2}$; The mass increases with increasing a due to $\gamma(a)$!

→ Metal, but with e-e interactions metal only at $a < a_c$ (Mott transition). The result depends on $\frac{U_c}{\gamma}$.

(ii) → Defs in the course.

→ $\Delta\left(\frac{1}{B}\right) = \frac{\hbar e}{m^* E_F}$ → $m^* = \frac{\hbar e}{E_F \Delta} = 1.1\times10^{-30}$ kg

→ $E_F = \frac{\hbar^2 k_F^2}{2m^*}$ → $k_F = \left(\frac{2m^* E_F}{\hbar^2}\right)^{1/2} = 6.5\times10^{9}\ m^{-1}$

→ $n = \frac{4\pi}{3}\frac{k_F^3}{(2\pi)^3} = 4.7\times10^{27}\ m^{-3}$, → $k_B T < \frac{\hbar e B}{m^*}$ → $T < 10.5$ K

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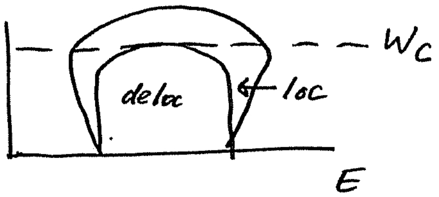
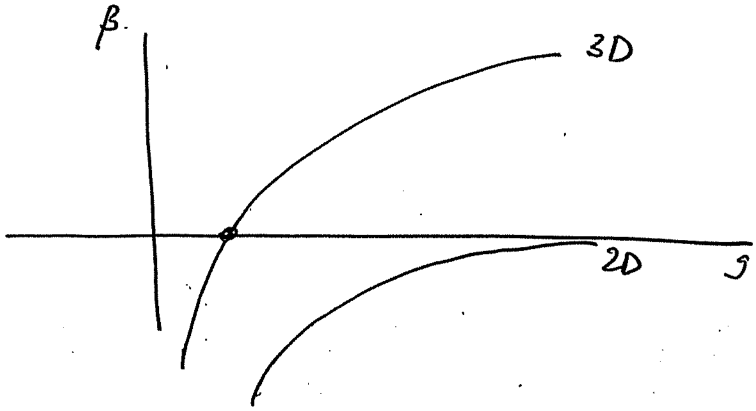
2. (i) → Defs in the course.

→ 

→ Band width increases, localisation occurs at the tails.

(ii) 

Large g:

3D $g \propto \frac{L^2}{L}$ → $\beta = 1$

2D $g \propto \frac{L}{L}$ → $\beta = 0$

Small g:

3D, 2D $g \propto \exp\left(-\frac{2L}{\xi}\right)$

$\beta = -\frac{2L}{\xi} = \frac{\ln g}{\text{const}}$

→ Experimentally, instead of changing L, T is changed

→ In the scaling theory there is no e-e intractions.

However, $r_s = \frac{U_c}{E_F} \propto \frac{m^*}{\sqrt{n}}$ can be large at low densities n.

Hints for Statistical Mechanics, 2007, PHYM421

1. Partition function Z is

$$Z = \sum_S e^{\mu\beta BS} = 2\cosh(x)$$
$$x = \beta\mu B, \quad p_\pm = \frac{1}{Z}e^{\pm x}$$

Hence,

$$\overline{\mu S} = \mu \sum_S S p_S = \mu \tanh x$$
$$\overline{E} = -\overline{\mu S}B = -\mu B \tanh x$$

Magnetisation, M, $M = \frac{N}{V}\overline{\mu S} = \frac{N\mu}{V}\tanh x$

At low T, $M \to \frac{N}{V}\mu$ as all spins are aligned. At high T, $x \to 0$ and

$$M \sim \frac{N}{V}\mu x$$
$$\chi \equiv M/H = \frac{N}{V}\frac{\mu^2\mu_0}{k_BT}$$

$$S = -Nk_B(p_+ \ln p_+ + p_- \ln p_-)$$

At low T, $p_+ \to 1$, $p_- \to 0$, hence $S \to 0$. $p_+ = \frac{e^x}{(e^x+e^{-x})} \sim 1 - e^{-2x}$. Hence $S \sim -k_B \ln(1-e^{-2x}) \sim k_B e^{-2\beta\mu B}$.

$$C_V = \left(\frac{\partial E}{\partial T}\right)_V = \frac{\mu^2 B^2}{k_B T^2}\mathrm{sech}^2(x) \sim x^2 \mathrm{sech}^2(x)$$

This vanishes at both high and low T and curve exhibits a maximum in between and differs from harmonic oscillators where C_V tends to zero at low and a constant at high T because instead of two, there are an infinite number of quantum states.

2. This is largely bookwork.

The density of particles

$$\rho = \frac{1}{V}\sum_k \overline{n}_k = \int \frac{N(E)dE}{e^{\beta(E-\mu)} \pm 1}$$

where $N(E)$ is density of states.

The average energy is

$$\overline{E} = V\int \frac{N(E)EdE}{e^{\beta(E-\mu)} \pm 1}$$

To calculate C_V for Fermions, a quick derivation of $E(T) - E(0)$ is to argue that at temperature T, $N(\mu)k_BT$ electrons are excited from $\mu - k_BT$ to $\mu + k_BT$ and hence energy change is product of these or

$$N(\mu)(k_BT)^2$$

Thus C_V is proportional to T.

3. (i) These definitions covered in the notes.

(ii) The Gibbs energy is

$$G = E - TS + pV$$
$$dG = dE - TdS - SdT + pdV + vdP$$
$$= -SdT + VdP$$

Hence,

$$\left(\frac{\partial G}{\partial T}\right)_p = -S$$
$$T\frac{d^2G}{dT^2} = -T\left(\frac{\partial S}{\partial T}\right)_p = -C_p$$

(iii) At the melting point, $\mu_l(T,p) = \mu_s(T,p)$ and hence as $G = N\mu$, $\Delta G(T,p) = 0$.

The latent heat is $L = T\Delta S$ or

$$-T\left(\frac{\partial \Delta G}{\partial T}\right)_p$$

(iv) From above,

$$-\frac{d^2\Delta G}{dT^2} = \frac{\Delta C_p}{T} = \frac{0.3}{T}$$
$$-\Delta S = \frac{d\Delta G}{dT} = -0.3\ln T + a$$
$$\Delta G = -0.3T\ln T + 0.3T + aT + b$$

Now at MP, $T = 1808$ K, $L = T\Delta S = 3670 = T(-a + 0.3\ln T)$ and hence $a = -3670/T - 0.3\ln 1808 = 0.220$

Also $\Delta G = 0$, hence $b + .520T - 0.3T\ln T = 0$ giving $b = 3127.6$.

Hence $\Delta G = -0.3T\ln T + 3127.6 + 0.520T$.

PHYM 422   2007.

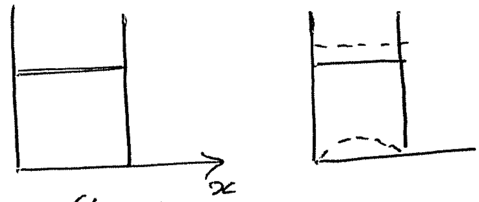
HINTS AND TIPS.

I. (i) See lecture notes.

$$\Delta E_n^{(2)} = \sum_{k \neq n} \frac{|V'_{kn}|^2}{E_n^{(0)} - E_k^{(0)}}$$

Example: Attractive interaction between electrons in SC.
(or effective mass in a solid in k.p method).

(ii)



1st term: $\Delta E^{(1)} = 0$ by symmetry   2nd term: $\Delta E^{(1)} > 0$.

2nd term: $$\Delta E^{(1)} = B \int_0^L \frac{2}{L} \sin^2\left(\frac{\pi}{L}x\right) \sin\left(\frac{\pi x}{L}\right) dx = \cdots = -\frac{3B}{2L}\frac{L}{\pi} \cos\frac{\pi x}{L}\Big|_0^L + \frac{B}{2L}\frac{L}{3\pi}\cos\frac{3\pi x}{L}\Big|_0^L$$

$$= \frac{8}{3}\frac{B}{\pi} > 0$$; Applicability: $\Delta E^{(1)} \ll E$, i.e. $\frac{8B}{3\pi} \ll \frac{\pi^2 \hbar^2}{2mL^2}$; while $\Delta E^{(2)} < 0$ for the ground state.

II. (i) $n = 2$; four-fold degenerate : 1s + 3p states
Name: the Stark effect, seen in splitting of spectral lines.

$E = 10^5 V/m$, $\Delta \mathcal{E} = 3ea_B E = 1.5 \times 10^{-3} eV \ll 13.6 \cdot \frac{1}{n^2} \sim 3.4 eV$

(ii) $\det(\hat{V} - \Delta \hat{I}) = 0 \rightarrow \Delta^2 - \frac{V_0^2}{4} \rightarrow \Delta_{1,2} = \pm\frac{V_0}{2}$, gap $= V_0$.

a) $\Delta_1 = \frac{V_0}{2}$ $-\frac{V_0}{2} b_1 + \frac{V_0}{2} b_2 = 0$, $b_2 = b_1$, $\varphi_1(x) = \frac{A}{\sqrt{L}}\left(e^{i\frac{\pi}{a}x} + e^{-i\frac{\pi}{a}x}\right)$.

b) $\Delta_2 = -\frac{V_0}{2}$, $\varphi_2(x) = \frac{A}{\sqrt{L}}\left(e^{i\frac{\pi}{a}x} - e^{-i\frac{\pi}{a}x}\right)$.

Normalisation: $$\frac{A^2}{L}\int_0^L (\psi_1^* \pm \psi_2^*)(\psi_1 \pm \psi_2) dx = A^2(1+1) = 1$$

$\rightarrow A = \frac{1}{\sqrt{2}}$ (using $\frac{1}{L}\int_0^L e^{i\frac{2\pi}{a}x} dx = 0$)
periodic function.

PHYM 422 2007

HINTS AND TIPS.

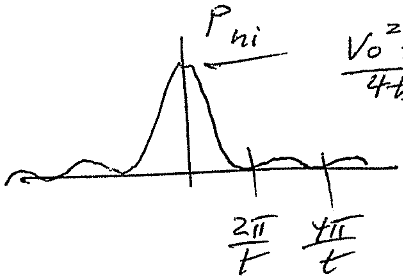
III. Meaning – see lecture notes.

$V_{ni}(t)$ – matrix element of the perturbation, $\omega_{ni} = \dfrac{E_n^{(0)} - E_i^{(0)}}{\hbar}$.

$$V_0 \cos\omega t = \frac{V_0}{2}\left(\exp i\omega t + \exp(-i\omega t)\right), \text{ then}$$

$$C_n^{(1)} = \frac{V_0}{2i\hbar}\int_0^t \left[\exp(i\omega t) + \exp(-i\omega t)\right] e^{i\omega_{ni}t}\,dt = \dots$$

$$= -\frac{V_0}{2\hbar}\,\frac{e^{i(-\omega+\omega_{ni})t} - 1}{-\omega+\omega_{ni}} \quad \text{and} \quad P_{ni} = \frac{V_0^2}{4\hbar^2}\,\frac{\sin^2\left(\frac{(\omega_{ni}-\omega)t}{2}\right)}{\left(\frac{\omega_{ni}-\omega}{2}\right)^2}.$$



→ There is NO energy conservation at a finite time of observation, exact resonance $\omega = \omega_{ni}$ at $t \to \infty$.

If t is doubled, the peak increases by 4 and the width decreases by 2.

IV. Definitions: see lecture notes.

$$\int_0^\infty V(r)\,\frac{\sin Kr}{Kr}\,r^2 dr = \frac{V_0}{K}\int_0^\infty e^{-\frac{r^2}{a^2}} \sin Kr\, r\,dr =$$

$$= V_0 a^3 \frac{\sqrt{\pi}}{4} \exp\left(-\frac{K^2 a^2}{4}\right).$$

Thus, $f(\theta) = -\dfrac{\sqrt{\pi}}{2}\dfrac{mV_0}{\hbar^2}a^3 \exp\left(-K^2a^2/4\right) \propto \underbrace{a}_{\text{length}}\dfrac{\overbrace{V_0}^{\text{energy}}}{\underbrace{\left(\frac{\hbar^2}{ma^2}\right)}_{\text{energy}}}$

$$f(\theta) \propto \exp\left(-a^2k^2 \sin^2\frac{\theta}{2}\right)$$

$f(\theta)$ is maximal at $\theta = 0$ → no change of the direction.

SOLUTION TO DEGREE EXAMINATION QUESTION

PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	PHY~~4~~M423
Name of module	Classical and Quantum Fluids
Date of examination	January 2007

1. Sheet stress at distance r from centre of circular pipe $\tau = -\frac{r}{2}\frac{\mathrm{d}p}{\mathrm{d}x}$

   Pressure drop across descending piston $\Delta p^* = 1.4\,\mathrm{MPa}$

   Drag coefficient for cylinder $C_\mathrm{D} = \dfrac{F_\mathrm{D}}{0.5\rho v_\infty^2\left(\pi D^2/4\right)}$

2. (i) N/A

   (ii) Heat dissipated must be less than:

$$\dot{Q} = A4h_\mathrm{K}T^3\Delta T = 50\times10^{-6}\,\mathrm{m}^2\times4\times10\,\mathrm{W\,m^{-2}\,K^{-4}}\times(30\,\mathrm{mK})^3\times2\,\mathrm{mK}$$

$$= 108\,\mathrm{pW} \quad\Rightarrow\quad V_\mathrm{max} = \sqrt{108\,\mathrm{pW}\times7\,\mathrm{k\Omega}} = 0.87\,\mathrm{mV}$$

3. Typical values: $C_\mathrm{v} = 0.98 \quad C_\mathrm{c} = 0.64 \quad C_\mathrm{d} = 0.63$

4. The superfluid fountain

$$\frac{\partial \mathbf{v}_\mathrm{s}}{\partial t} = S\nabla T - \frac{1}{\rho}\nabla P = 0 \quad\Rightarrow \Delta T = \frac{gh}{S} = \frac{9.8\,\mathrm{m\,s^{-2}}\times0.1\,\mathrm{m}}{85\,\mathrm{J\,kg^{-1}\,K^{-1}}} = 12\,\mathrm{mK}$$

$$v = \sqrt{2gh} = \sqrt{2\times9.8\,\mathrm{m\,s^{-2}}\times0.1\,\mathrm{m}} = 1.4\,\mathrm{m\,s^{-1}}$$

$$\dot{Q} = Av C\Delta T = 1\,\mathrm{mm}^2\times1.4\,\mathrm{m\,s^{-1}}\times\left(53\,\mathrm{J\,kg^{-1}\,K^{-1}}\times145\,\mathrm{kg\,m^{-3}}\right)\times12\,\mathrm{mK} = 130\,\mu\mathrm{W}$$

SOLUTION TO DEGREE EXAMINATION QUESTION

PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	**PHY4423**
Name of module	**Classical and Quantum Fluids**
Date of examination	**January 2007**

1. Sheet stress at distance r from centre of circular pipe $\tau = -\frac{r}{2}\frac{\mathrm{d}p}{\mathrm{d}x}$

   Pressure drop across descending piston $\Delta p^{*} = 1.4\,\mathrm{MPa}$

   Drag coefficient for cylinder $C_\mathrm{D} = \dfrac{F_\mathrm{D}}{0.5\rho v_\infty^2\left(\pi D^2/4\right)}$

2. (i) N/A

   (ii) Heat dissipated must be less than:

$$\dot{Q} = A4h_\mathrm{K}T^3\Delta T = 50\times10^{-6}\ \mathrm{m}^2\times4\times10\ \mathrm{W\,m^{-2}\,K^{-4}}\times(30\ \mathrm{mK})^3\times2\ \mathrm{mK}$$
$$= 108\ \mathrm{pW}$$

3. Typical values: $C_\mathrm{v} = 0.98$ $\quad C_\mathrm{c} = 0.64$ $\quad C_\mathrm{d} = 0.63$

4. The superfluid fountain

$$\frac{\partial \mathbf{v}_\mathrm{s}}{\partial t} = S\nabla T - \frac{1}{\rho}\nabla P = 0 \quad \Rightarrow \Delta T = \frac{gh}{S} = \frac{9.8\ \mathrm{m\,s^{-2}}\times0.1\ \mathrm{m}}{85\ \mathrm{J\,kg^{-1}\,K^{-1}}} = 12\ \mathrm{mK}$$

$$v = \sqrt{2gh} = \sqrt{2\times9.8\ \mathrm{m\,s^{-2}}\times0.1\ \mathrm{m}} = 1.4\ \mathrm{m\,s^{-1}}$$

$$\dot{Q} = Av C\Delta T = 1\ \mathrm{mm}^2\times1.4\ \mathrm{m\,s^{-1}}\times\left(53\ \mathrm{J\,kg^{-1}\,K^{-1}}\times145\ \mathrm{kg\,m^{-3}}\right)\times12\ \mathrm{mK} = 130\ \mu\mathrm{W}$$

PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	**PHYM424**
Name of module	**Semiconductors and Heterstructures**
Date of examination	**June 2007**

1. $E_0 = 2.5$ meV

2. Acceptor concentration $= 9.6 \times 10^{-9}$ cm^{-2}

4. $r_1 = 12.15$ nm $\quad E_1 = 4.6$ meV

Hints for Relativity and Cosmology 2007, PHYM432

1. Use Lorentz transformation to show that equation of world line of a particle undergoing SHM changes is

$$x^1(p) = a\cos p, \quad x^2(p) = a\sin p$$
$$x^0(p) = c\frac{p}{\omega}, \quad x^3(p) = 0$$

while for a frame moving along x-axis with speed $c/2$, is

$$x^{1'} = \gamma(x^1 - x^0/2) = \gamma(a\cos p - cp/2\omega)$$
$$x^{2'} = x^2 = a\sin p$$
$$x^{0'}(p) = \gamma(x^0 - x^1/2) = \gamma(cp/\omega - a\cos p/2), \quad x^3(p) = 0$$

If light bulbs are at $-l$ and l then time observer detects light is l/c. Light line is $(ct, l-ct)$ and (ct,-l+ct) in frame S and and (ct', x') in s'. For light from right bulb

$$x' = \gamma(x - vt) = \gamma(l - (c+v)t)$$
$$ct' = \gamma(ct - v(l - ct)/c)$$

This reaches origin when $t = l/(c+v)$ for which $t' = \gamma l(1 - v/c))$
For left bulb, light line is $x' = \gamma(-l + ct - vt)$, $t' = \gamma(t - (ct - l)v/c^2)$ and this reaches origin when $t = l/(c-v)$ and at time $t' = \gamma l(1 + v/c)$.

2. 3-velocity of light moving towards fixed origin from fixed star at angle α with x-axis, is w $= (-c\cos\alpha, -c\sin\alpha, 0)$. Suppose an observer is moving along x-axis with speed w. Then velocity of light in frame S' is

$$w'_x = -\frac{(c\ \cos\ \alpha + v)}{1 + v/c\ \cos\ \alpha)}$$
$$w'_y = -\frac{c\ \sin\ \alpha}{\gamma(1 + v/c\ \cos\ \alpha)}$$
$$w'_z = 0$$

or $c(-\cos\ \alpha', -\sin\ \alpha', 0)$
where

$$\tan\ \alpha' = \frac{\sin\ \alpha}{\gamma(\cos\ \alpha + v/c)}$$

Recalling $-c\cos(\alpha') = -\frac{(c\ \cos\ \alpha+v)}{1+v/c\ \cos\ \alpha)}$ and maximum value of α is $\pi/2$ for which $\cos\alpha' = v/c$.

3. Book work

4. Schwarzschild metric for planar orbits is

$$ds^2 = Bc^2dt^2 - \frac{dr^2}{B} - r^2d\phi^2$$
$$B = 1 - 2GMc^2/r = 1 - r_s/r$$

Clearly goes to flat space-time when $c = \infty$. Variables have usual meaning: t, r, ϕ are time, radial coordinate and angular coordinate. For $r < r_s$ terms change sign so that length and time become interchanged.

Take mass of Earth to be 5.98 $\times 10^{24}$ kg. $G = 6.67\ 10^{-8}$ dyne cm^2 g^{-2}, $c = 3\ 10^{10}$, $MG/c^2 = .443$ cm or $r_s = .88$ cm

PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	PHYM434
Name of module	Signal processing
Date of examination	June 2007

1(i) All have complex poles at $s = \frac{w_0}{2}\left[-\frac{1}{\varphi} \pm j\sqrt{4-\frac{1}{\varphi^2}}\right]$

(a) no zeros (b) zero at origin (c) 2 zeros at origin (d) zeros at $\pm j w_0$

(e) zeros at $s = \frac{w_0}{2}\left[\frac{1}{\varphi} \pm j\sqrt{4-\frac{1}{\varphi^2}}\right]$

(ii) (a) $|H| = \dfrac{1}{\sqrt{1 + w^4/w_0^4}}$, 2nd order lowpass (etc.)

2. $g_1(w) = 2j \sin\frac{w\tau}{2}$, $g_2(w) = -4\sin^2\frac{w\tau}{2}$

$g_3(w) = -8j\sin^3\frac{w\tau}{2}$, $g_4(w) = 16\sin^4\frac{w\tau}{2}$

f_5 is time integral of f_4, so $g_5 = \frac{1}{jw} g_4$

3 (ii) Weights add to 144, so normalise by $\times \frac{1}{144}$

Column totals are $\frac{1}{36}$ $\frac{17}{72}$ $\frac{17}{36}$ $\frac{17}{72}$ $\frac{1}{36}$

Line profile is 180 180 176 146 112 146 176 180 180

Lowpass filter.

4. see notes.