UNIVERSITY OF EXETER - SCHOOL OF PHYSICS
SOLUTION TO EXAMINATION QUESTION

Module Number	PHYM401
Year of Examination	2006
Question Number	1-2
Name of Setter	AKS
Initials of Checker	HINTS AND SOLUTIONS

(i) → $\omega_c \tau >> 1$, where $\omega_c = \frac{eB}{m^*}$ and τ is the momentum relaxation time

Hint: compare the period of orbiting with time between two scattering events.

→ $\frac{\rho_{300}}{\rho_{4.2}}$ is larger for pure materials ($\rho_{4,2}$ is determined by impurity scattering)

→ $l = V_F \tau$, $E_F = \frac{mV_F^2}{2}$; therefore τ can be found and thus $\frac{eB\tau}{m} = 1$.

Answer: $B >> 3.4$ mT.

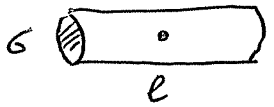
→ 

→ SdH effect is oscillations $\rho(B)$ due to Landau quantisation.
$K_B T \ll \frac{\hbar eB}{m^*}$ – low temperatures to observe the effect.

(ii) → $n = 0, 1 \ldots$ is a quantum number; → The condition $\frac{R_c}{\lambda_F} >> 1$ is satisfied for $n >> 1$.

(i) → Screening of Coulomb interaction; Pauli principle.

→ e-e scattering does not change the drift velocity and hence does not control ρ.

→ 

Condition: $n \times \sigma l = 1$ gives $l = \frac{1}{n\sigma}$.

→ Important relation: $\sigma' = \sigma_0 \left(\frac{K_B T}{E_F}\right)^2$, hence $\sigma'_{1K} = 7.3 \times 10^{-24}$ cm^2 and $l = \frac{1}{n\sigma} = 1,4$ cm, which is much larger than Lee.

(ii) → Total number of states in a B.Z. is 2N.

→ Half occupancy gives a metal and full occupancy – an insulator.
The first can be an insulator due to Mott's transition (e-e interactions)
The second – a metal if 2N electrons are in fact in 1st and 2nd BZs (like Ca).

→ Important parameters: in the first case ~~E_g~~ $\frac{U_c}{\gamma}$ – band width, in the second – E_g in the short direction of the B.Z.

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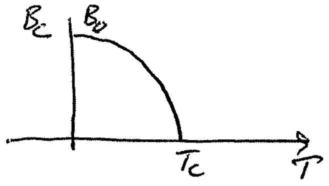
→ Cooper pairs : two electrons opposite spin and momentum K.
They have spin zero and thus are bosons.

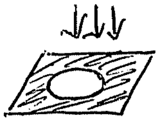
→ The Meissner effect : B=0 inside the SC.

→ 

Important relation : $B_c(T) = B_0\left(1-\left(\frac{T}{T_c}\right)^2\right) = 52.8\,mT$ at $T=4.2K$

→ BCS : $\frac{\Delta_0}{k_B T_c} = 1.76$; $h\nu < 2\Delta_0$ gives $\nu < 5.2\times10^{11}\,Hz$.

→ Supereconductor is a poor thermal conductor → poor thermal contact with the bath.

→ 

A hole in a SC ; Magnetic field will penetrate through the hole in accordance with $\Phi = B\underset{\substack{\uparrow \\ \text{area of hole}}}{A} = \underbrace{\frac{h}{2e}}_{\text{SC flux quantum}} n$ $n = 1,2\ldots$

→ main properties of SC here : macroscopic coherence of the SC state ; 2e in the flux shows indeed a Cooper pair.

→ Definition : $D(\omega,k) = \varepsilon_0\varepsilon(\omega,k)E(\omega,k)$

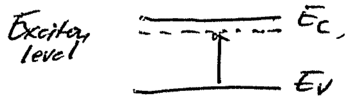
→ $\varepsilon(\omega,0)$ describes responce of electron gas at $\lambda\to\infty$.
$\varepsilon(0,k)$ — " — electrostatic screening.

→ Plasma : equal concentration of -ive and +ive charges, of which at least one is mobile !

→ (Lecture notes) $\varepsilon(\omega) = \frac{D(\omega)}{\varepsilon_0 E(\omega)} = 1 + \frac{P(\omega)}{\varepsilon_0 E(\omega)}$ — main relations.

→ Plasmon : a quantum of plasma oscillations in a metal, can be excited by an electron beam. (as quantised energy loss).

→ Exciton : an electron-hole pair interacting by Coulomb attraction.



Seen in experiment as optical absorption below the interband threshold.

Hints for Statistical Mechanics PHYM421/PHYM501

1. (i) Use $F = E - TS$ and differentiate.

   (ii) Recall that for a system in equilibrium, the free energy is minimized. Therefore, find the solution of
   $$\frac{dF}{dM} = 0 = (a(T - T_c)M + bM^3)$$
   which yields lowest value of F.

   Show for $T > T_c$,
   $$\begin{aligned} F = F_0 - \frac{a^2}{2b}(T - T_c)^2 + \frac{a^2}{4b}(T - T_c)^2 &= F_0 - \frac{a^2}{4b}(T - T_c)^2 \\ S = -\frac{dF}{dT} &= -\frac{dF_0}{dT} + \frac{a^2}{2b}(T - T_c) \\ C_V = T\frac{dS}{dT} &= -T\frac{d^2F_0}{dT^2} + \frac{a^2T}{2b} \end{aligned}$$
   and hence deduce discontinuity in C_V.

2. Recall the Boltzmann definition of entropy in terms of p_r amd write down the thermodynamic variable corresponding to $\frac{\partial S}{\partial E}$.

   If $p_r = 1/\Omega$, write S in terms of Ω where Ω is statistical weight.

   For the problem, Stirlings approximation shows
   $$\Omega = \frac{(N - 1 + fN)!}{(N - 1)!(fN)!} = \frac{(N - 1 + fN)^{N-1+fN}}{(N - 1)^{N-1}(fN)^{fN}}$$
   and use this to show $S/k_B = (1 + f)N \ln(1 + f) - fN \ln f$.

   Hence use $1/T = \frac{\partial S}{\partial E}$ to deduce $f = 1/(e^{\hbar\omega/k_BT} - 1)$.

3. First section is in the notes.

   Recall to find the chemical potential, we have that the total number of particles satisfies
   $$N = \int dE \frac{g(E)}{e^{\beta(E-\mu)} - 1} = \frac{N_0}{E_0}\int_0^\infty \frac{dE}{e^{\beta(E-\mu)} - 1} = -\frac{N_0}{\beta E_0}\ln(1 - e^{\beta\mu})$$
   Then show
   $$\begin{aligned} 1 - e^{\beta\mu} &= e^{-\beta E_0 N/N_0} \\ \mu &= kT\ln(1 - e^{-\beta E_0 N/N_0}) \end{aligned}$$
   For any value of N, there is always a solution to $\mu < 0$, hence Bose-Einstein condensation does not occur.

4. Recall,

$$\Phi = E - TS - \mu N$$

and for this problem,

$$\begin{aligned}\Phi = n(E_c-\mu) + k_BTn\ln(n/N_ce) + N_d\{f(E_d-\mu) + \\ k_BTf\ln f + k_BT(1-f)\ln 1 - f\} + (N_v-p)(E_v-\mu) + k_BTp\ln(p/N_ve) \\ \frac{\partial\Phi}{\partial n} = E_c - \mu + k_BT\ln(n/N_c) = 0 \\ \text{or } n = N_ce^{-(E_c-\mu)/k_BT} \\ \frac{\partial\Phi}{\partial f} = N_d(E_d - \mu + k_BT\ln f - k_BT\ln(1-f)) = 0 \\ \text{or } f = \frac{1}{1+e^{(E_d-\mu)/k_BT}} \\ \frac{\partial\Phi}{\partial p} = -E_v + \mu + k_BT\ln p/N_v = 0 \\ \text{hence } p = N_ve^{-(\mu-E_v)/k_BT}\end{aligned}$$

Use $fN_d \sim n >> p$, at low temp to show that $\mu = \frac{1}{2}(E_c - E_d) + k_BT\ln(N_d/N_c)$ at low temperatures and $\frac{(E_c-E_v)}{2} + k_BT\ln(N_v/N_c)$ at high temperature.

UNIVERSITY OF EXETER - SCHOOL OF PHYSICS
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Module Number	PHYM 422
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1.

(i) → $\psi_n^{(0)}$ are unperturbed W.Fs , $C_{nk}^{(1)} = \dfrac{V'_{kn}}{E_n^{(0)} - E_k^{(0)}}$ ← matrix elements. ← unperturbed eigenenergies

$$\Delta E_n = V'_{nn} = \int_{-\infty}^{\infty} |\psi_n|^2 V' dx$$

(ii) → $V' = \frac{1}{2} m\omega^2 \frac{x^3}{L}$ → $\Delta E_0^{(1)} = 0$ as the integrand is an odd function, therefore high-order corrections are important!

→ $\Delta E_0^{(2)} = \sum_{k\neq 0} \dfrac{|V'_{k0}|^2}{E_0^{(0)} - E_k^{(0)}}$ = (only two non-zero elements!) = $-\dfrac{11}{32} \dfrac{\hbar\omega}{L^2\alpha}$.

2.

(i) → $\varphi_{nk'}^{(0)} = \sum_{k=1}^{N} b_{k'k} \psi_{nk}^{(0)}$, with equation to determine the coefficients $b_{k'k}$:

$$(\hat{V} - \Delta \hat{I})\hat{u} = 0$$

→ $\det \begin{pmatrix} -\Delta & iV_0 \\ -iV_0 & -\Delta \end{pmatrix} = 0$ → $\Delta_{1,2} = \pm V_0$

→ wavefunctions : $b_2 = -ib_1$ → $\varphi_1 = \frac{1}{\sqrt{2}}(\psi_1 - i\psi_2)$

$b_2 = ib_1$ → $\varphi_2 = \frac{1}{\sqrt{2}}(\psi_1 + i\psi_2)$

(ii) a) $S=0$ (spin part is antisymmetric)

$$\rightarrow \psi = \frac{1}{\sqrt{2}} \frac{2}{L} \left\{ \sin\frac{\pi x_1}{L} \cdot \sin\frac{2\pi x_2}{L} + \sin\frac{\pi x_2}{L} \cdot \sin\frac{2\pi x_1}{L} \right\}$$

b) $S=1$ (spin part is symmetric)

$$\rightarrow \psi = \frac{1}{\sqrt{2}} \frac{2}{L} \left\{ \sin\frac{\pi x_1}{L} \cdot \sin\frac{2\pi x_2}{L} - \sin\frac{\pi x_2}{L} \cdot \sin\frac{2\pi x_1}{L} \right\}$$

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SOLUTION TO EXAMINATION QUESTION

Module Number	PHY M 422
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3.

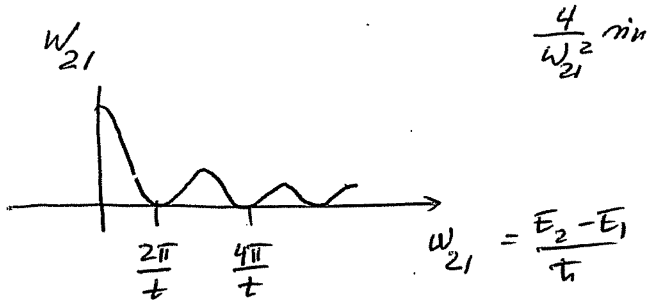
→ $W_{mn}(t) = \frac{1}{\hbar^2} \left| \int_0^t V_{mn}(t) e^{i\omega_{mn}t} dt \right|^2$, $\omega_{mn} = \frac{E_m - E_n}{\hbar}$.

$V_{mn}(t) = \int \varphi^*_m(x) V(t,x) \varphi_n(x) dx$.

→ $V(t) = -F_0 x$ gives $V_{2,1} = \frac{16 F_0 L}{9\pi^2}$

Then $W_{21} = \frac{1}{\hbar^2}\left(\frac{16 F_0 L}{9\pi^2}\right)^2 \left| \underbrace{\int_0^t e^{i\omega_{21}t} dt}_{\frac{4}{\omega_{21}^2}\sin^2\frac{\omega_{21}t}{2}} \right|^2$, $\omega_{21} = \frac{3}{2}\frac{\pi^2\hbar}{mL^2}$

→



4.

→ Lecture notes for definition of $f(\theta)$.

→ Born approximation is valid if the scattering potential is weak and $\psi_s \ll \psi(0)$

→ $f(\theta) = -\frac{2mA}{\hbar^2 K_0} \int_0^\infty \sin(Kr) dr = -\frac{2mA}{\hbar^2 \{2k^2(1-\cos\theta) + \gamma^2\}}$

→ Total cross-section $\sigma = \int |f(\theta)|^2 d\Omega = 2\pi \int_0^\pi |f(\theta)| \sin\theta d\theta$

→ The screening length $\lambda = \frac{1}{\gamma}$

With increasing γ , λ decreases and scattering becomes less efficient.

SOLUTION TO DEGREE EXAMINATION QUESTION

PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	**PHY4423**
Name of module	**Classical and Quantum Fluids**
Date of examination	**January 2006**

1. Use conservation of mass, i.e. $\rho A_1 \mathbf{v}_1 = \rho A_2 \mathbf{v}_2$ to argue that $v_2 = 2v_1$

Start from $Q = v_1 A_1 = v_2 A_2$ and $p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$
hence $Q = \sqrt{\dfrac{2(p_1 - p_2)}{\rho\left(1/A_c^2 - 1/A_1^2\right)}}$

2. (i) Refer to lecture notes

(ii)

$$\dot{Q}_{\text{cooling}} + \dot{Q}_{\text{leak}} = \dot{N}_3 T_{\text{mc}}^2 84\ \text{J mol}^{-1}\ \text{K}^{-2} = 20\ \mu\text{W} + \dot{Q}_{\text{leak}}$$

$$\dot{Q}_{\text{leak}} = \dot{N}_3 T_{\text{base}}^2 84\ \text{J mol}^{-1}\ \text{K}^{-2} = \dot{N}_3 \times 225 \times 10^{-6} \times 84\ \text{J mol}^{-1}$$

$$\dot{N}_3\left((100\ \text{mK})^2 - (15\ \text{mK})^2\right) \times 84\ \text{J mol}^{-1}\ \text{K}^{-2} = 20\ \mu\text{W}$$

$$\Rightarrow \quad \dot{N}_3 = 24.4\ \mu\text{mol s}^{-1} \quad \text{and} \quad \dot{Q}_{\text{leak}} = 0.46\ \mu\text{W}$$

The base temperature is high but the cooling power is only slightly reduced => problem must be extra heat leak. Using above equations, the heat leak is equivalent to 5 μW into the mixing chamber. A reduced value of $\dot{N}_3$ would have a much big effect on the cooling power.

3. (i) $C_{\text{D}} = \dfrac{F_{\text{D}}}{0.5\rho v_{x\infty}^2 A} = \dfrac{3\pi d\mu v_{x\infty}}{0.5\rho v_{x\infty}^2 0.25\pi d^2} = \dfrac{24}{Re}$

(ii) Refer to handouts.

4. The normal boiling point of ^{4}He is 4.2 K so

$$p_0 = 1\ \text{bar} \times \exp\left(\frac{7.2\ \text{K}}{4.2\ \text{K}}\right) = 5.5\ \text{bar}$$

$$p(100\ \text{mK}) = 5.5\ \text{bar} \times \exp\left(\frac{-7.2\ \text{K}}{100\ \text{mK}}\right) = 3 \times 10^{-31}\ \text{bar}$$

$$pV = nk_{\text{B}}T \quad \text{so} \quad n = \frac{3 \times 10^{-31}\ \text{bar} \times 10^5\ \text{N m}^{-2}\ \text{bar}^{-1}}{1.38 \times 10^{-23}\ \text{J K}^{-1} \times 0.1\ \text{K}} = 0.02\ \text{m}^{-3}$$

PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	PHYM424
Name of module	Semiconductors and Heterostructures
Date of examination	June 2006

1. (ii) $\Delta E_g^c = 0.49\,\mathrm{eV}$

$E_1^e - E_1^h = 2.3\times10^{-19}\ \mathrm{J} = 1.4\,\mathrm{eV}$

PHYM 428
General Problems

FOR DISCUSSION WITH PERSONAL TUTORS, THEREFORE NO HINTS &TIPS ARE PROVIDED

Hints for Relativity and Cosmology 2006, PHYM432

1. Use Lorentz transformation

$$x' = \gamma(x - vt), \quad t' = \gamma(t - vx/c^2),$$

and let $\cosh\phi = \gamma$ to show

$$\begin{aligned}\sinh\phi &= \sqrt{\cosh^2\phi - 1}\\ &= \sqrt{\gamma^2 - 1} = \gamma v/c\end{aligned}$$

Recall for a particle moving with speed w,

$$\begin{aligned}c^2 d\tau^2 &= c^2dt^2 - dx^2 - dy^2 - dz^2\\ &= c^2dt^2(1 - w^2/c^2)\end{aligned}$$

Use Lorentz transformation

$$\begin{aligned}dx^{1\prime} &= \gamma(dx^1 - (v/c)dx^0)\\ dx^{0\prime} &= \gamma(dx^0 - (v/c)dx^1)\end{aligned}$$

and $dx^1 = (w_x/c)dx^0$ and $dx^{1\prime} = (w'_x/c)dx^{0\prime}$ to show

$$\begin{aligned}w'_x &= \frac{(w_x - v)}{1 - vw_x/c^2}\\ w'_y &= \frac{w_y}{\gamma(1 - vw_x/c^2)}\end{aligned}$$

Applying this we get

$$\tan\alpha = \frac{w'_y}{w'_x} = -\frac{u}{\gamma v}$$

2. Use composition of velocities, to deduce

$$\begin{aligned}\frac{du'}{dt} &= \frac{du}{dt}/(1 - vu/c^2) + \frac{du}{dt}(u - v)v/(c^2(1 - vu/c^2)^2)\\ &= \frac{du}{dt}\frac{(1 - v^2/c^2)}{(1 - uv/c^2)^2}\end{aligned}$$

Finally, set $v = u$ and $u' = 0.$ to deduce

$$\frac{du'}{dt'} = \gamma^3\frac{du}{dt} = \frac{d\gamma u}{dt}$$

If LHS is constant $\equiv \alpha$, then, integrate twice to obtain $\alpha t = \gamma u$

$$u = \frac{\alpha t}{\sqrt{1 + \alpha^2 t^2/c^2}}$$

and

$$x = (c^2/\alpha)\sqrt{1 + \alpha^2 t^2/c^2}$$

If photon emitted a distance x behind particle, then in time t travels a distance ct but by then particle having proper acceleration α is at distance $x = \sqrt{c^2t^2 + c^4/\alpha^2}$ and hence photon never catches particle.

3. This is covered in notes.

4. First sections are book work.

   In early Universe, gravitation terms dominates k in Friedmann eqn. Hence choose $k = 0$ and rewrite Friedman equations:

$$\frac{d\rho c^2 R^3}{dt} = -3R^2 p\dot{R}, \quad p = \rho c^2/3$$

$$\dot{\rho}/\rho = -4\dot{R}/R \;\; or \;\; \rho R^4 = \rho_0 R_0^4 \text{ .Now as } \rho c^2 = aT^4, \;\; T = T_0\frac{R_0}{R}$$

$$\text{Friedmann first eqn} \;\; \dot{R}^2 = \frac{8\pi G\rho R^2}{3}, \;\; R\dot{R} = \sqrt{8\pi G\rho_0 R_0^4/3}$$

$$\text{Hence} \;\; \frac{1}{2}R^2(t) = \sqrt{\frac{8\pi G\rho_0 R_0^4}{3}}t$$

PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	**PHYM433**
Name of module	**MRI & Biophysics**
Date of examination	**June 2006**

1. Descriptive material covered in lectures.

2. Descriptive material covered in lectures.

3 (i) $\frac{1}{T_2^*} - \frac{1}{T_2} = 1{\cdot}34 \times 10^8 \, \Delta B_z$

Also $\exp\left[-TE\left(\frac{1}{T_2^*} - \frac{1}{T_2}\right)\right] = 0{\cdot}2$

$\Rightarrow \frac{1}{T_2^*} - \frac{1}{T_2} = \frac{1{\cdot}61}{TE}$

} Combine to give $\Delta B_z = 1{\cdot}00 \times 10^{-6}\,T$

(ii) Shift equivalent to $5{\cdot}25 \times 10^{-6}\,T$,

Use gradient strength to convert to displacement 0·875 mm.

4. $w = \gamma B_z = \gamma(B_0 + G_x x) = w_0 + \gamma G_x x$

t maps onto position in k-space, e.g. $k_x = \gamma G_x t$

Last part: principal echoes towards centre of k-space where signal is strongest.