PROBLEMS

4 Special Chapters

4.1 The set of numbers i, $-i$, 1 and -1, with ordinary multiplication form a group.

(a) Write down the multiplication table for this group.

(b) What is the unit element of this group?

(c) What is the inverse element of -1?

(d) Show that this group is cyclic.

(e) Show that this group is Abelian.

4.2 Two matrices

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

with ordinary multiplication rule for matrices form a group.

(a) Write down the multiplication table for this group.

(b) What is the unit element of this group?

(c) What is the inverse element of B?

(d) Show that this group is Abelian.

4.3 Consider a network consisting of N sites.

(a) $N/6$ sites of this network are blocked. What is the probability of a randomly selected site being unblocked?

(b) Formulate a definition of a percolation threshold of the site problem and find a percolation threshold in a square network consisting of four sites ($N = 4$).

4.4 Find the probability $P_2(x)$ for a randomly chosen site to belong to a cluster consisting of not less than 2 unblocked sites, where x is unblocked site density,

(a) for a square plane lattice;

(b) for a simple cubic lattice;

(c) for an arbitrary lattice in which each site has z nearest neighbours.

4.5 For the bond percolation problem find the probability $P^b(x)$ of the randomly chosen site to belong to an infinite cluster for $1 - x << 1$:

(a) for a square plane lattice;

(b) for a triangular lattice;

(c) for a honeycomb lattice.