PROBLEMS

2 Gamma and Beta Functions

2.1 Prove that

(a) $\Gamma(n+1) = n\Gamma(n)$, $n > 0$;

(b) $\Gamma(n+1) = n!$, $n = 1,2,3,\ldots$

2.2 Prove that $\Gamma(1/2) = \sqrt{\pi}$.

2.3 Prove that

(a) $B(m,n) = B(n,m)$;

(b) $B(m,n) = 2 \int_{0}^{\pi/2} \sin^{2m-1}\theta \, \cos^{2n-1}\theta \, d\theta$.

2.4 Prove that $B(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

2.5 Given $\displaystyle\int_{0}^{\infty} \frac{x^{p-1}}{1+x} \, dx = \frac{\pi}{\sin p\pi}$, show that

$$\Gamma(p)\Gamma(1-p) = \frac{\pi}{\sin p\pi} \text{ where } 0 < p < 1.$$