# PHYSICS EXAMINATION PROBLEMS
# SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

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| **Module Code** | **PHY3140** |
| **Name of module** | **Methods of theoretical physics** |
| **Date of examination** | **May/June 2010** |

1. (i) E.g., classification of vibrational modes or selection rules for optical transitions.

(a) $\underline{\underline{C}} = \underline{\underline{A}}\,\underline{\underline{B}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. (b)

| | *A* | *B* | *C* |
|---|---|---|---|
| *A* | *B* | *C* | *A* |
| *B* | *C* | *A* | *B* |
| *C* | *A* | *B* | *C* |

(c) $\underline{\underline{E}} = \underline{\underline{C}}$. (d) Multiplication table is symmetric ($\underline{\underline{A}}\,\underline{\underline{B}} = \underline{\underline{B}}\,\underline{\underline{A}}$), therefore the group is Abelian. All elements can be expressed as $\underline{\underline{A}}^k$, where $k = 1,2,3$, $\Rightarrow$ the group is cyclic. (e) $\underline{\underline{A}}^{-1} = \underline{\underline{B}}$.

(ii) (a) $\int_0^{2\pi} \frac{d\theta}{a + \sin\theta} = \frac{2\pi}{\sqrt{a^2 - 1}}$ . Hint: Substitute $z = \exp(i\theta)$ and use contour integration.

(b) $\int_0^{+\infty} \frac{dx}{\left(x^2 + 1\right)^3} = \frac{3\pi}{16}$. Hint: Contour integration and the residue formula for a third-order pole.

2. (i) $\lambda = -1$. $w(x, y) = 2xy + A$, where $A$ is a real number. $f(z) = z^2 + iA$.

(ii) $E = \left(\frac{\pi}{2\sqrt{2}\gamma}\right)^{5/3} \left(\frac{\alpha\hbar^{10}}{m^5}\right)^{1/6} \left(n + \frac{1}{2}\right)^{5/3} \approx 1.32 \times \left(\frac{\alpha\hbar^{10}}{m^5}\right)^{1/6} \left(n + \frac{1}{2}\right)^{5/3}$.

Here $\gamma = \int_0^1 \sqrt{1 - t^{10}}\, dt \approx 0.94$.

3. (i) Taylor series: $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n+1)n!}$.[Integrate the integrand's Taylor expansion.]

Asymptotic expansion: $\operatorname{erf}(x) = 1 - \frac{1}{\sqrt{\pi}} \left[ \frac{1}{x} - \frac{1}{2x^3} + \frac{1 \cdot 3}{2^2} \frac{1}{x^5} - O\left(\frac{1}{x^7}\right) \right]$. [Use by-parts integration]

The error function is used for data analysis.

(ii) $P = (N - M)/N$. Bookwork (definition). $x_c(4) = 5/12$.

4. $A = 2b^{3/2}$. $E = \left(\frac{3}{2}\right)^{5/3} \left(\frac{\hbar^2 F^2}{m}\right)^{1/3}$.