Gas of N particles in a cubic container, side length L.

If $N/n_{\text{states}} << 1$ we have a "classical gas".

If $N/n_{\text{states}} \sim 1$ we have a "quantal gas".

Behaviour of a quantal gas is strongly determined by the Pauli Exclusion Principle:

Any number of bosons can occupy a given quantum state but only one fermion can occupy a given quantum state.

Half-integer spin particles (e.g. e, p, n) are "Fermions".
Integer-spin particles (e.g. γ, phonon) are "Bosons".

Consider <ε>, the mean energy of each of the N particles in the container.

$$\langle \varepsilon \rangle = \frac{(p_{\text{mean}})^2}{2m}$$

$n_{\text{states}} \approx$ vol. of momentum space enclosed by an octant of radius p_{mean} / volume occupied by one state

$$n_{\text{states}} \approx \frac{1}{8}\frac{4}{3}\pi p_{\text{mean}}^3 \left(\frac{L}{\hbar\pi}\right)^3 \sim V\left(\frac{p_{\text{mean}}}{h}\right)^3 \sim V\left(\frac{1}{\lambda_{\text{deBroglie}}}\right)^3$$

Hence

$$\frac{N}{n_{\text{states}}} \sim \frac{N}{V}\lambda_{\text{deBroglie}}^3 \sim \left(\frac{\lambda_{\text{deBroglie}}}{\text{mean particle spacing}}\right)^3$$

Hence a gas becomes quantal when the mean inter-particle spacing becomes comparable with the particles' de Broglie wavelength.

e.g. H at STP, molar volume 22.4×10^{-3} m^3.
Mean spacing = $(22.4\times10^{-3}/6\times10^{23})^{1/3}$ = 3×10^{-9} m.
At room temp,

$$\lambda_{\text{deBroglie}} = \frac{h}{\sqrt{2m\langle\varepsilon\rangle}} = \frac{h}{\sqrt{2mk_B T}} = \frac{6.6\times10^{-34}}{\sqrt{2\times1.6\times10^{-27}\times1.4\times10^{-23}\times300}} = 0.2\times10^{-9}\text{ m}$$

Only 1 in 1000 states typically occupied hence it is safe to treat as a "classical" gas i.e. rules for filling states are unimportant.

e.g. gas of conduction electrons in a metal, density typically 10^{28} m^{-3}.

Mean spacing = $(10^{28})^{-1/3}$ = 0.5×10^{-9} m.

$$\lambda_{\text{deBroglie}} = \frac{6.6\times10^{-34}}{\sqrt{2\times10^{-30}\times1.4\times10^{-23}\times300}} = 6\times10^{-9}\text{ m}$$

Hence conduction electrons form a quantal gas i.e. the rules for filling states are important. e's are fermions so only one particle can occupy a given quantum state.

As $T\rightarrow0$ the electrons will crowd into the lowest available energy level. Unlike a classical ensemble they cannot all move into the ground state, because only one particle is allowed per state. Instead they will fill all available states up to some maximum energy, the Fermi energy E_F.