PHY2201 Summary Sheet 18

Single particle mass m confined to a cubic container (3-D ∞ potential well) side length L.

Describe particle via a wavefunction $\Psi(x,y,z)$ satisfying the energy eigenvalue equation :-

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Psi(x,y,z)+V(x,y,z)\Psi(x,y,z)=E\Psi(x,y,z)$$

Solutions E and $\Psi(x,y,z)$ are the energy eigenvalues and stationary states of the particle.

Boundary conditions:
$\Psi(0,y,z) = \Psi(L,y,z) = \Psi(x,0,z) = \Psi(x,L,z) = \Psi(x,y,0) = \Psi(x,y,L) = 0$

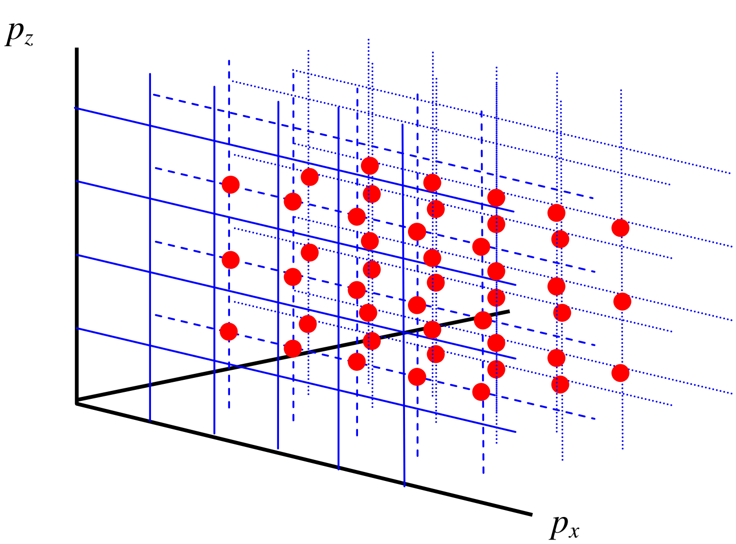
Solution :-

$$\Psi_{n_x,n_y,n_z}(x,y,z)=A\sin\left(n_x\frac{\pi x}{L}\right)\sin\left(n_y\frac{\pi y}{L}\right)\sin\left(n_z\frac{\pi z}{L}\right)$$

$$E_{n_x,n_y,n_z}=\frac{h^2}{8mL^2}\left(n_x^2+n_y^2+n_z^2\right)$$

$p_x = n_x\frac{\hbar\pi}{L}$ (ditto y,z) $n_x = 1, 2, 3 \ldots$ (ditto n_y, n_z)

Momentum space picture :-



Allowed states form a cubic lattice, lattice constant $\frac{\hbar\pi}{L}$.

Hydrogen atom at room temp confined to 1m^3.

$$\langle \varepsilon \rangle = \frac{\langle p^2 \rangle}{2m} \approx k_B T$$

$$p \approx \sqrt{2mk_B T}$$

$$\approx \sqrt{2\times1.6\times10^{-27}\times1.4\times10^{-23}\times300} \approx 3.7\times10^{-24}\,\mathrm{kg\,m\,s^{-1}}$$

$$\frac{\hbar\pi}{L} = \frac{10^{-34}\times3}{1} = 3\times10^{-34}\,\mathrm{kg\,ms^{-1}}$$

Hence momentum states are very finely spaced ⇨ can often treat as forming a continuum.