PHY2201 Summary Sheet 9

Ensemble of N gas atoms. Outer electron can reside in a "ground-state" energy level, or in an excited state, 1 eV above this. At 1000 K, what fraction of atoms lie in the excited state, relative to the ground-state?

Boltzmann distribution:

$$\frac{n_i}{N} = \frac{\exp(-\varepsilon_i/k_B T)}{\sum_{i=0}^{\infty}\exp(-\varepsilon_i/k_B T)}$$

n_i is no. of systems occupying a state of energy ε_i, when ensemble of N such systems is in thermal equilibrium at temp T. For a 2-level system, energies ε_1, ε_2, relative occupancy of these levels is given by

$$\frac{n_2}{n_1} = \frac{\exp(-\varepsilon_2/k_B T)}{\sum_{i=0}^{\infty}\exp(-\varepsilon_i/k_B T)} \frac{\sum_{i=0}^{\infty}\exp(-\varepsilon_i/k_B T)}{\exp(-\varepsilon_1/k_B T)} = \exp(-(\varepsilon_2-\varepsilon_1)/k_B T)$$

$$\boxed{\frac{n_2}{n_1} = \exp(-\Delta\varepsilon/k_B T)}$$

with $\Delta\varepsilon = \varepsilon_2 - \varepsilon_1$

Useful "rule of thumb":

At room temperature (300K) the thermal energy $k_B T$ is 25 meV

Hence at 1000 K the thermal energy is 25 meV × 1000/300.

$\therefore\ n_2/n_1 = \exp(-1/(25*10\times10^{-3}/3)) = \exp(-12) = 5\times10^{-6}$

$\underline{n_2/n_1 = 6\times10^{-6}}$

Cool the gas to 300 K:-

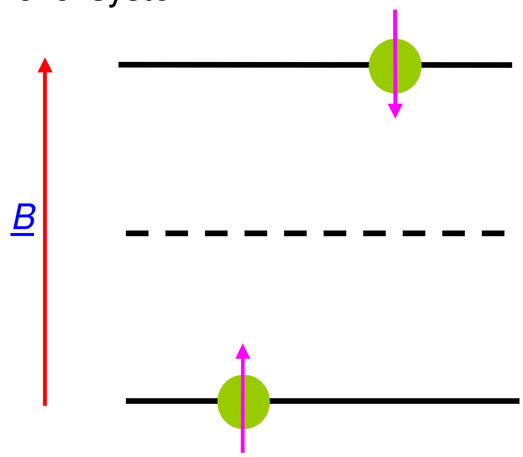
$n_2/n_1 = \exp(-1/(25*\times10^{-3})) = \exp(-40) = 5\times10^{-6}$

$\underline{n_2/n_1 = 4\times10^{-18}}$

very strong T-dependence.

Ensemble of protons, magnetic moment μ, in external magnetic field B. Magnetostatic potential energy = $+\mu B$ if proton spin anti-parallel to field, $-\mu B$ if proton spin parallel to field.

Simple 2-level system:



In equilibrium, what is the net imbalance between spin "aligned" ($n_\uparrow$) and spin "anti-aligned" ($n_\downarrow$) protons (i.e. what is the fractional magnetization) at room temperature?

$$\frac{n_\uparrow - n_\downarrow}{n_\uparrow + n_\downarrow} = \frac{1 - n_\downarrow / n_\uparrow}{1 + n_\downarrow / n_\uparrow}$$

$$= \frac{1 - \exp\left(-(\varepsilon_1 - \varepsilon_0)/(k_B T)\right)}{1 + \exp\left(-(\varepsilon_1 - \varepsilon_0)/(k_B T)\right)} = \frac{1 - \exp\left(-2\mu B/(k_B T)\right)}{1 + \exp\left(-2\mu B/(k_B T)\right)}$$

Proton $\mu = 1.46\times10^{-26}$ JT^{-1}. B = 1 T (typically), T = 300 K.

Hence $2\mu B = 2.9\times10^{-26}$ J. $k_B T = 1.38\times10^{-23}\times300$ J $= 3.12\times10^{-21}$ J.

Since $2\mu B << k_B T$ $\quad \exp\left(-2\mu B/(k_B T)\right) \cong 1 - 2\mu B/(k_B T)$

$$\frac{n_\uparrow - n_\downarrow}{n_\uparrow + n_\downarrow} \cong \frac{1 - \left(1 - 2\mu B/(k_B T)\right)}{1 + \left(1 - 2\mu B/(k_B T)\right)} \cong \frac{\mu B}{k_B T}$$

Hence at 300K and 1 T, the net imbalance of proton spins is 1.46×10^{-26} / 3.12×10^{-21} J

<u>= 4.7×10^{-6}</u>

i.e. a very <u>small</u> imbalance.