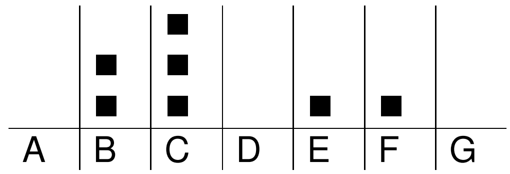
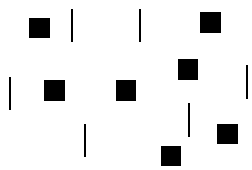
PHY2201 Summary Sheet 8

In total, there are 1,716 possible ways to share 7ε amongst 7 identical systems. To calculate this directly, consider 7 distinguishable heaps A, B, C… G. How many ways can we distribute 7 identical bricks among these? One possibility IS



Equivalent problem. Consider a pile of 7 bricks and 6 partitions (all indistinguishable).



If we draw objects (bricks or partitions) at random, each distinct sequence of bricks and partitions corresponds to exactly one possible distribution of 7 bricks amongst 7 heaps e.g. the possible arrangement noted above corresponds to the sequence

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If bricks and partitions are indistinguishable, no. of ways equals

$$\frac{(7+6)!}{7!6!} = 1{,}716$$

Hence in general, we can distribute N packets of energy over k systems in

$$\frac{(N+k-1)!}{N!(k-1)!} \text{ ways}$$

The fundamental postulates of statistical mechanics

1. An ensemble of identical but distinguishable systems can be described completely by specifying its “microstate”. The microstate is the most detailed description of an ensemble that can be provided. For an ideal gas of N particles in a container, it involves specifying $6N$ co-ordinates, the position and velocity of all N particles. For the example of Boltzmann energy sharing, it involves specifying the energy level occupied by each individual system.

2. Physically we observe only a corresponding “macrostate”, specified in terms of macroscopically observable quantities. A macrostate for an ideal gas is specified fully by a few observable quantities such as pressure, temperature, volume, entropy etc. For the example of Boltzmann energy sharing, a macrostate is specified fully by the occupancies of the various energy levels e.g. [0,7,0,0,0,0,0,0…] is a macrostate of equal energy sharing.

3. If we observe an ensemble over time, random perturbations ensure that all accessible microstates will occur with equal probability. Hence probability of a macrostate occurring =

$$\frac{\text{no. of microstates corresponding to that macrostate}}{\text{total number of microstates}}$$

4. The macrostate with the highest probability of occurrence corresponds to the equilibrium state.

Boltzmann distribution

Maximise

$$\Omega = \frac{N!}{\prod_{i=0}^{\infty} n_i!}$$

subject to the constraints $\sum_{i=0}^{\infty} n_i = N \quad \sum_{i=0}^{\infty} n_i \varepsilon_i = U$

using Lagrange Undetermined Multipliers (see supplementary sheet).

Solution:

$$\frac{n_i}{N} = \frac{\exp(-\beta\varepsilon_i)}{\sum_{i=0}^{\infty} \exp(-\beta\varepsilon_i)}$$ (with β undetermined)

Assigning $\beta = 1/k_B T$

$$\boxed{\frac{n_i}{N} = \frac{\exp(-\varepsilon_i/k_B T)}{\sum_{i=0}^{\infty} \exp(-\varepsilon_i/k_B T)}}$$

Boltzmann distribution

NB n_i/N is the probability that a state of energy ε_i is occupied by a member of an ensemble which is in thermal equilibrium at temperature T.

THE BOLTZMANN DISTRIBUTION IS THE MOST IMPORTANT RESULT IN THIS COURSE !!